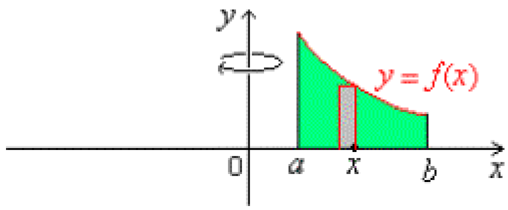


Fig. 1

The colored region is revolved about the y-axis.

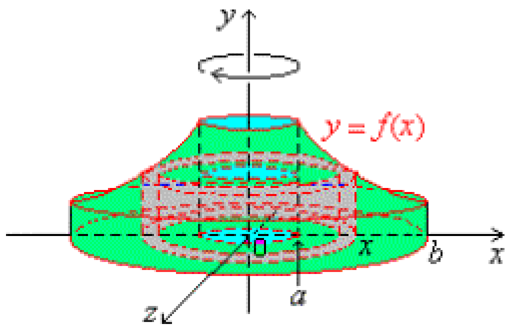


Fig. 2

The solid of revolution and a cylindrical shell.

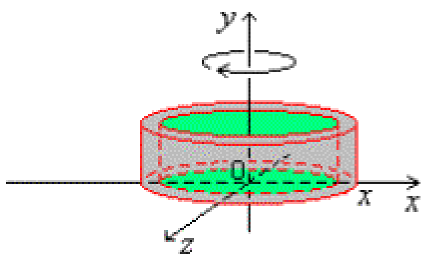
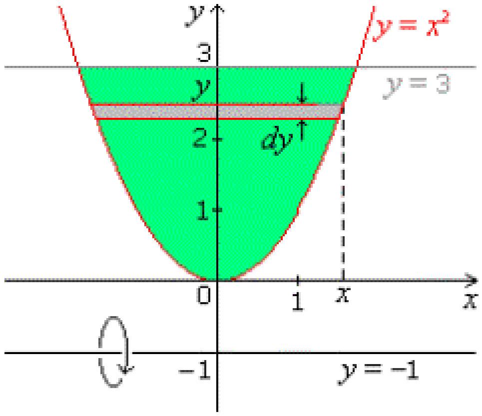


Fig. 3

The cylindrical shell is reproduced here for clarity.

Find the volume of the solid generated by revolving the plane region bounded by $y = x^2$ and $y = 3$ about the line $y = -1$. Now use the shell method to find that volume.



Plane region bounded by

$y = x^2$ and $y = 3$ is

revolved about line $y = -1$.

Thickness of shell: dy

Average radius: $y-(-1) = y+1$

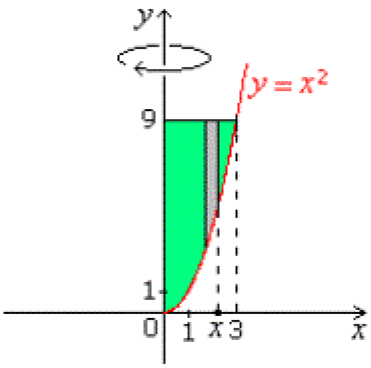
Altitude: $\sqrt{y} - \sqrt{y} = 2\sqrt{y}$

Volume: $2\pi(y+1)\cdot 2\sqrt{y}\, dy$

$$V = 4\pi\int_0^3 (y+1)\sqrt{y}dy$$

$= \frac{112\sqrt{3}\pi}{5}$ cubic units.

Use the shell method to find the volume of the solid generated by revolving the plane region bounded by $y = x^2$, $y = 9$, and $x = 0$ about the *y*-axis.

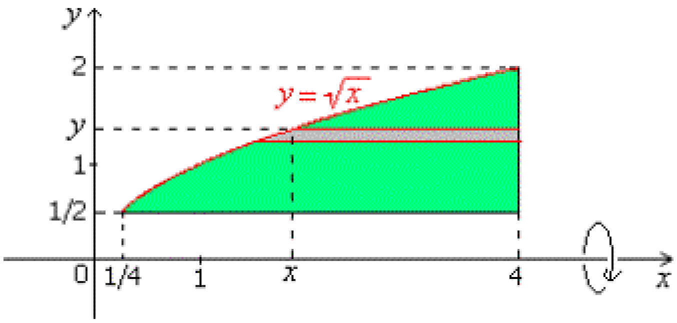


Thickness of shell:	dx
Average radius:	x
Altitude:	$9 - x^2$
Volume:	$2\pi \cdot x(9 - x^2)$

$$V = 2\pi \int_0^3 x(9 - x^2)dx$$

$\frac{81\pi}{2}$ cubic units.

Use the shell method to find the volume of the solid generated by revolving the plane region bounded by $y = \sqrt{x}$, $y = \frac{1}{2}$, and $x = 4$ about the *x*-axis.



Thickness of shell: dy

Average radius: y

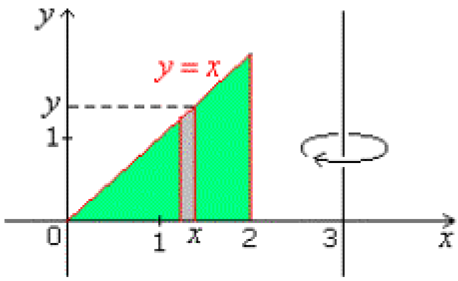
Altitude: $4 - y^2$

Volume: $2\pi \cdot y(4 - y^2)$

$$V = 2\pi \int_{\frac{1}{2}}^{2} y(4 - y^2)dy$$

$\frac{225\pi}{32}$ cubic units.

Use the shell method to find the volume of the solid generated by revolving the triangular region bounded by $y = x$, $y = 0$, and $x = 2$ about the line $x = 3$.



Thickness of shell:	dx
Average radius:	$3 - x$
Altitude:	x
Volume:	$2\pi \cdot (3 - x)x$

$$V = 2\pi \int_0^2 (3 - x)x \, dx$$

$\frac{20\pi}{3}$ cubic units.