Math 182 Real Test 1 Print Name ______________________

McDonald Spring 2016 B

You must show all work!

1. Find the **general solution** of the differential equation.

$$\frac{dy}{dx} = 5x^4 - 9x^3$$

$$dy = \left(5x^4 - 9x^3\right)dx$$

$$\int 1\,dy = \int \left(5x^4 - 9x^3\right)dx$$

$$y = \frac{5x^5}{5} - \frac{9x^4}{4} + C$$

$$\boxed{y = x^5 - \frac{9}{4}x^4 + C}$$

2. Find the **general solution** of the differential equation.

$$y' = \frac{4x}{y}$$

$$y\,dy = 4x\,dx \quad \text{Separate Variables}$$

$$\int y\,dy = \int 4x\,dx$$

$$\frac{1}{2}y^2 = \frac{4}{2}x^2 + C_1$$

$$\frac{1}{2}y^2 = 2x^2 + C_1$$

$$2\left(\frac{1}{2}y^2\right) = 2\left(2x^2 + C_1\right)$$

$$y^2 = 4x^2 + 2C_1 \quad Let\ C = 2C_1$$

$$\boxed{y^2 = 4x^2 + C}$$

3. Find the **general solution** to the differential equation. Then find **the particular solution** with initial condition $y(1) = 5$.

$$y' - 3x^2 y = 0$$

$$\frac{dy}{dx} = 3x^2 y$$

$$\frac{1}{y}\frac{dy}{dx}dx = \frac{3x^2 y}{y}dx$$

$$\frac{1}{y}dx = 3x^2 dx \quad \text{Separate Variables}$$

$$\int \frac{1}{y}dx = \int 3x^2 dx \quad \textit{Now,} \text{intergrate Both sides.}$$

$$\ln|y| = \frac{3x^3}{3} + C_1$$

$$\ln|y| = x^3 + C_1$$

$$\ln|y| = x^3 + C_1$$

$$e^{\ln|y|} = e^{x^3 + C_1} = e^{x^3} e^{C_1} \text{ let } C = e^{C_1}$$

$$\boxed{|y| = Ce^{x^3} \quad \text{General Solution}}$$

Plug in (1,5)

$$|5| = Ce^{1^3} = ce$$

$$5 = Ce \Rightarrow C = \frac{5}{e}$$

$$y = \frac{5}{e}e^{x^3} = 5e^{-1}e^{x^3} = 5e^{x^3 - 1}$$

$$\boxed{y = 5e^{x^3 - 1} \quad \text{Particular Solutions}}$$

We could try using the first order technique. It works either way much to my surprise!

$$y' - 3x^2 y = 0$$

$$y' + P(x)y = Q(x) \;\; so \;\; P(x) = -3x^2 \;\; and \;\; Q(x) = 0$$

$$u(x) = e^{\int -3x^2\,dx} = e^{-x^3}$$

$$y = \frac{1}{u(x)}\int Q(x)u(x)\,dx = \frac{1}{e^{-x^3}}\int 0(e^{-x^3})\,dx = \frac{1}{e^{-x^3}}(0 + C)$$

$$\boxed{y = Ce^{x^3} \quad \text{General Solution}}$$

plug in (1,5)

$$5 = Ce^{1^3} = Ce \Rightarrow C = \frac{5}{e}$$

$$y = \frac{5}{e}e^{x^3} = 5e^{-1}e^{x^3} = 5e^{x^3 - 1}$$

$$\boxed{y = 5e^{x^3 - 1} \quad \text{Particular Solution}}$$

4. Find the general solution of the differential equation $y' + 2xy = 10x$.

$$y' + P(x)y = Q(x) \;\; so \;\; P(x) = 2x \;\; and \;\; Q(x) = 10x$$

$$u(x) = e^{\int 2x\,dx} = e^{x^2}$$

$$y = \frac{1}{u(x)}\int Q(x)u(x)\,dx = \frac{1}{e^{x^2}}\int 10xe^{x^2}\,dx = \frac{10}{e^{x^2}}\int xe^{x^2}\,dx$$

$$\int xe^{x^2}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$Let \; u = x^2, \; du = 2x\,dx \Rightarrow \tfrac{1}{2}du = x\,dx$$

$$y = \frac{10}{e^{x^2}}\int e^u\,du = \frac{10}{e^{x^2}}\left(\frac{1}{2}e^{x^2} + C\right) = 5 + Ce^{-x^2}$$

$$\boxed{y = 5 + Ce^{-x^2} \quad \text{General Solution}}$$

You can separate this too. $\dfrac{dy}{dx} = 10x - 2xy = 2x(5 - y) \Rightarrow \boxed{\dfrac{1}{5 - y}dy = 2x\,dx \;\; or \;\; \dfrac{1}{y - 5}dy = -2x\,dx}$

5. Find the general solution of the differential equation $y' + 6y = e^{6x}$.

$$y' + P(x)y = Q(x) \;\; so \;\; P(x) = 6 \; and \; Q(x) = e^{6x}$$

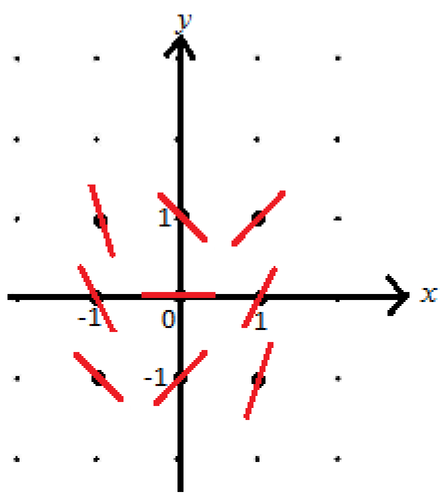
$$u(x) = e^{\int 6\,dx} = e^{6x}$$

$$y = \frac{1}{u(x)} \int Q(x)u(x)\,dx = \frac{1}{e^{6x}} \int e^{6x}e^{6x}\,dx = \frac{1}{e^{6x}} \int e^{12x}\,dx$$

$$y = \frac{1}{e^{6x}} \underbrace{\int \frac{1}{12} e^{u}\,du}_{let\, u=12x,\, du=12\,dx} = \frac{1}{e^{6x}}\left(\frac{1}{12}e^{12x} + C\right) = \frac{1}{12}e^{6x} + Ce^{-6x}$$

$$\boxed{y = \frac{1}{12}e^{6x} + Ce^{-6x} \text{ General Solution}}$$

6. Sketch the **slope field** for the differential equation. $y' = 2x - y$
Use the nine points that are bold and larger.
Can this be separated? Explain?



x	y	$y' = 2x - y$
-1	-1	$-2+1=-1$
-1	0	$-2+0=-2$
-1	1	$-2-1=-3$
0	-1	$0+1=1$
0	0	$0-0=0$
0	1	$0-1=-1$
1	-1	$2+1=3$
1	0	$2-0=2$
1	1	$2-1=1$

7. Write the differential equation for the following statement.

The rate of change of the variable ***y*** is proportional to the value of ***y***.
Let *y* be a function of time *t*.

Show that the solutions are in the form of $y = Ce^{kt}$

$$y' = ky$$
$$\frac{dy}{dt} = ky$$
$$\frac{1}{y}\,dy = k\,dt$$
$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$e^{\ln|y|} = e^{kt+C_1}$$
$$e^{\ln|y|} = e^{kt}e^{C_1} \qquad let\;\; C = e^{C_1}$$
$$|y| = Ce^{kt} \quad \text{if } y > 0, \text{then}$$
$$y = Ce^{kt}$$

Note: We need a new constant. $C \neq e^C$ **There is no value of *C* that makes this true?**

$5 \neq e^5 \;\; or \;\; 3 \neq e^3$ we let get a new constant, $C = e^{C_1}$

8. Assume that $y = Ce^{kt}$ is a general solution to a Radioactive Decay differential equation with the initial conditions: *y* = 50 when *t* = 0. Let *y* represent the mass in grams of carbon. The half-life of carbon isotopes is 5715 years.

a) Find the particular equation and
b) then find how long it would take for 50 grams to decay to 10 grams.

$$y = Ce^{kt} \quad If\; t = 0, when\; y = 50.$$
$$50 = Ce^{k(0)} = Ce^0 = C(1) = C,\; so\; \boxed{C = 50.}$$
$$y = 50e^{kt}.\;\; \text{Now find } k.$$

$$y = 50e^{kt} \quad \text{there is 25 grams left after } 5715\,years.$$
$$25 = 50e^{k(5715)} \Rightarrow \frac{1}{2} = e^{5715k}$$
$$\ln\left(\frac{1}{2}\right) = \ln e^{5715k} \Rightarrow 5715k = \ln\left(\tfrac{1}{2}\right)$$
$$\boxed{k = \frac{\ln\left(\frac{1}{2}\right)}{5715}}$$

$$y = 50e^{kt} \quad where\;\; k = \frac{\ln\left(\frac{1}{2}\right)}{5715}.$$
$$10 = 50e^{kt}$$
$$\frac{1}{5} = e^{kt} \Rightarrow \ln\left(\tfrac{1}{5}\right) = kt$$
$$\boxed{t = \frac{\ln\left(\frac{1}{5}\right)}{\frac{\ln\left(\frac{1}{2}\right)}{5715}} = \frac{5715\ln\left(\frac{1}{5}\right)}{\ln\left(\frac{1}{2}\right)} \approx 13{,}269.82\; years}$$

9. Use Euler's method to approximate the particular solution to $\frac{dy}{dx} = x + y$ passing through the point (0,1). Use 4 steps of 0.5.

x	$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$
$x_0 = 0$	$y_0 = 1$
$x_1 = 0.5$	$y_1 = 1 + .5(0+1) = 1.5$
$x_2 = 1$	$y_2 = 1.5 + .5(.5+1.5) = 2.5$
$x_3 = 1.5$	$y_3 = 2.5 + .5(1+2.5) = 4.25$
$x_4 = 2$	$y_4 = 4.25 + .5(1.5+4.25) = 7.125$

10. Find the general solution of the differential equation $\frac{dy}{dx} = 5x$.

$$dy = 5x\,dx$$

$$\int dy = \int 5x\,dx$$

$$\boxed{y = \frac{5}{2}x^2 + C}$$