Math 182 Sample Test 2 Name _______________

McDonald

You must show all work! 6 points each

Evaluate the integral. Use integration by parts

1. $\int xe^{-3x}\,dx$

$$\int xe^{-3x}\,dx = uv - \int v\,du = x\left(-\frac{1}{3}e^{-3x}\right) - \int -\frac{1}{3}e^{-3x}\,dx$$

$u = x \qquad dv = e^{-3x}dx$

$du = dx \qquad v = -\frac{1}{3}e^{-3x}$

$$= \frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\,dx = \frac{1}{3}xe^{-3x} + \frac{1}{3}\left(-\frac{1}{3}e^{-3x}\right) + C$$

$$\boxed{= \frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C}$$

Just set up using partial fractions. Do not evaluate. Ask if you are confused.

2. $\int \frac{3x^2+4x-6}{(x-3)^2(x^2+9)}\,dx$

Set up for partial fraction. $\boxed{\frac{3x^2+4x-6}{(x-3)^2(x^2+9)} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{Cx+D}{x^2+9}}$

Do NOT do this.!!

$$\int \frac{3x^2+4x-6}{(x-3)^2(x^2+9)}\,dx = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{Cx+D}{x^2+9}$$ **Not equal!**

The LHS side is the integral. We haven't taken the integral of the RHS yet.

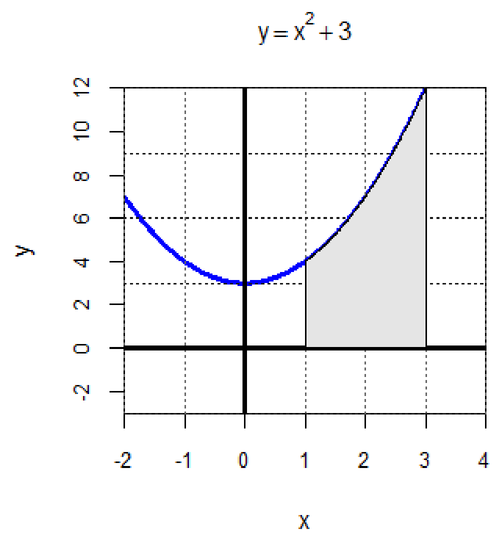
3. Find the **area** of the region bounded by the graphs: Please Graph!

$y = x^2 + 3,\ y = 0,\ x = 1$ and $x = 3$

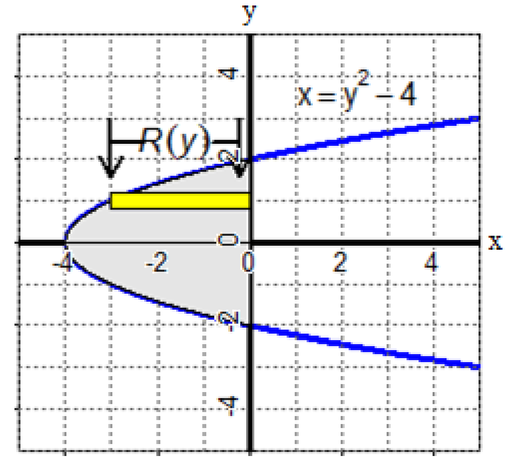
$$A = \int_1^3 \left(x^2+3-0\right)dx = \frac{1}{3}x^3 + 3x\Big|_1^3 = \frac{1}{3}3^3 + 3(3) - \left(\frac{1}{3}1^3 + 3(1)\right)$$

$$= 9 + 9 - \frac{1}{3} - 3 = 15 - \frac{1}{3} = \frac{45-1}{3} = \frac{44}{3}$$

$$\boxed{\frac{44}{3}\text{units}^2}$$



4. Find the **volume** of the solid formed by revolving the region bounded by $x = y^2 - 4, x = 0$ about the line y-axis. Use **Any Method**.



$$\boxed{V = \frac{512\pi}{15} units^3}$$

The Washer (or Disk) method

$R(y) = y^2 - 4, \quad r(y) = 0$

$$V = \pi \int_{-2}^{2} (y^2 - 4)^2 \, dy = \pi \int_{-2}^{2} (y^4 - 8y^2 + 16) \, dy$$

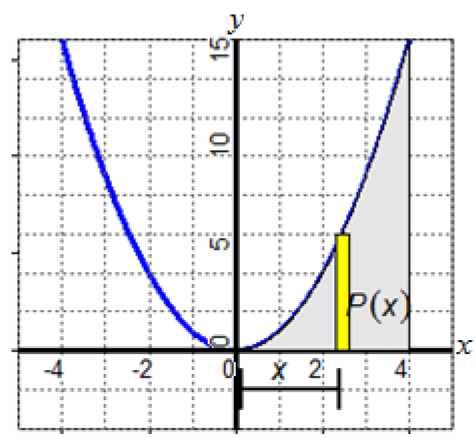
$$= \pi \left[\frac{1}{5} y^5 - \frac{8}{3} y^3 + 16y \right]_{-2}^{2}$$

$$= \pi \left[\left(\frac{1}{5}(2)^5 - \frac{8}{3}(2)^3 + 16(2) \right) - \left(\frac{1}{5}(-2)^5 - \frac{8}{3}(-2)^3 + 16(-2) \right) \right]$$

$$= \pi \left[\frac{256}{15} - \left(-\frac{256}{15} \right) \right] = \pi \left[\frac{256}{15} + \frac{256}{15} \right] = \frac{512\pi}{15}$$

5. - 6. Find the **volume** of the solid formed by revolving the region bounded by $y = x^2$, $y = 0$, and $x = 4$

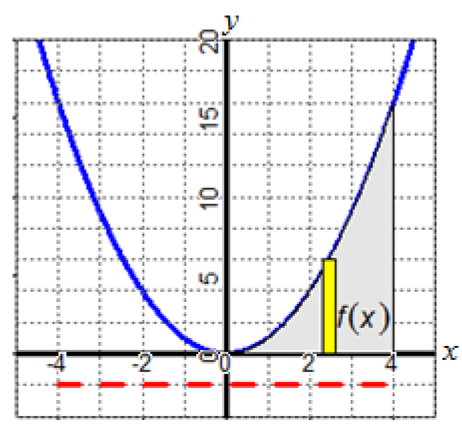
5. about the line y-axis. Use **Shell Method**.



$$V = 2\pi \int_0^4 x \cdot x^2 dx = 2\pi \frac{1}{4} x^4 \Big|_0^4 = \frac{\pi}{2} 4^4 = 128\pi$$

$$\boxed{V = 128\pi \, unit^3}$$

6. about the line $y = -2$. Use **Disk Method**.



$$\boxed{V = \frac{4352\pi}{15} unit^3}$$

$R(x) = f(x) - (-2) = x^2 + 2; \; r(x) = 0 - (-2) = 2$

$$V = \pi \int_0^4 \left[(x^2 + 2)^2 - (2)^2 \right] dx = \pi \int_0^4 \left[x^4 + 4x^2 + 4 - 4 \right] dx$$

$$= \pi \left[\frac{1}{5} x^5 + \frac{4}{3} x^3 \right]_0^4 = \pi \left[\frac{4^5}{5} + \frac{4^3}{3} - 0 \right] = \frac{4352\pi}{15}$$

Evaluate the integral.

7. $\int \sqrt{4-x^2}\,dx$

Let $x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$\sin\theta = \frac{x}{2}$

$\sqrt{4-x^2} = \sqrt{4-(2\sin\theta)^2} = \sqrt{4-4\sin^2\theta} = \sqrt{4(1-\sin^2\theta)}$

$= \sqrt{4\cos^2\theta} = 2\cos\theta$

$$\int \sqrt{4-x^2}\,dx = \int (2\cos\theta)(2\cos\theta)\,d\theta = \int 4\cos^2\theta\,d\theta$$

Use $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$.

$$4\int \cos^2\theta\,d\theta = 4\int \frac{1+\cos(2\theta)}{2}\,d\theta = 2\int \left(1+\cos(2\theta)\right)d\theta = 2\left(\theta + \frac{1}{2}\sin(2\theta)\right) + C$$

$$= 2\theta + \sin(2\theta) + C = 2\theta + 2\sin\theta\cos\theta + C$$

$$= 2\sin^{-1}\left(\frac{x}{2}\right) + (2)\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2} + C$$

$$\boxed{= 2\sin^{-1}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{2} + C}$$

Evaluate the integral.

8. $\int \sin^3 x\cos^2 x\,dx$

Let $u = \cos x$

$du = -\sin x\,dx$

$$\int \sin^3 x\cos^2 x\,dx = \int \sin x\;\sin^2 x\;\cos^2 x\,dx = \int \left(1-\cos^2 x\right)\cos^2 x\sin x\,dx$$

$$= -\int (1-u^2)u^2\,du = -\int (u^2-u^4)\,du = \int (u^4-u^2)\,du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

$$\boxed{= \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}$$

Evaluate the following integrals

9. $\int \sin^2 x \, dx$

Just memorize these two identities: $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$ and $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$$\int \sin^2 x \, dx = \int \frac{1-\cos(2x)}{2} dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right) dx = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C$$

$$\boxed{= \frac{1}{2}x - \frac{1}{4}\sin(2x) + C}$$

10. $\int \ln x \, dx$ Hint: Use integration by parts.

Let $u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x} dx$ $\quad v = x$

$$\int \ln x \, dx = uv - \int v du = (\ln x)x - \int x\left(\frac{1}{x}\right) dx = x \ln x - \int 1 dx$$

$$\boxed{= x \ln x - x + C}$$

Note: There is no reason to factor $x(\ln x - 1) + C$. Factoring is the opposite of simplifying.

Evaluate the following integrals

11. $\int \frac{1}{x^2 - 4x + 8} dx$ Looks like arctan x?

Let $u = x - 2$ $\quad a = 2$

$du = dx$

$$\int \frac{1}{x^2 - 4x + 8} dx = \int \frac{1}{x^2 - 4x + 4 + 4} dx = \int \frac{1}{(x-2)^2 + 4} dx = \int \frac{1}{(x-2)^2 + 2^2} dx$$

$$= \int \frac{1}{u^2 + 2^2} du = \boxed{\frac{1}{2}\tan^{-1}\left(\frac{x-2}{2}\right) + C}$$

12. $\int \frac{x+4}{x^2-4x}\,dx$ Use partial fractions.

$$\frac{x+4}{x(x-4)}=\frac{A}{x}+\frac{B}{x-4}$$

$$\left[\frac{x+4}{x(x-4)}\right]\big(x(x-4)\big)=\left[\frac{A}{x}+\frac{B}{x-4}\right]\big(x(x-4)\big)$$

$$x+4=A(x-4)+Bx$$

Let $x=0,$	*Let* $x=4,$
$0+4=A(0-4)+B(0)$	$4+4=A(4-4)+B(4)$
$4=-4A$	$8=4B$
$A=-1$	$B=2$

$$\int \frac{x+4}{x^2-4x}\,dx=\int\left(\frac{-1}{x}+\frac{2}{x-4}\right)dx=\boxed{-\ln|x|+2\ln|x-4|+C=\ln\left|\frac{(x-4)^2}{x}\right|+C=\ln\frac{(x-4)^2}{|x|}+C}$$

Anyone of these three forms is correct.

Evaluate the integral. Memorized answer or prove if you must.

13. $\int \frac{2}{x\sqrt{x^2-4}}\,dx=\int \frac{2}{x\sqrt{x^2-2^2}}\,dx=2\left(\frac{1}{2}\sec^{-1}\left(\frac{|x|}{2}\right)\right)+C=\boxed{\sec^{-1}\left(\frac{|x|}{2}\right)+C}$ Done!!

Bonus Points for using trig substitution. State trig substitution.

13. Continue… Bonus Points for using trig substitution. State trig substitution.

$$\int \frac{2}{x\sqrt{x^2-4}}\,dx=$$

$$x=2\sec\theta \qquad \sec\theta=\frac{x}{2}$$

$$dx=2\sec\theta\tan\theta\,d\theta \qquad \theta=\sec^{-1}\left(\frac{x}{2}\right)$$

$$\sqrt{(2\sec\theta)^2-4}=\sqrt{4\sec^2-4}=\sqrt{4(\sec^2\theta-1)}=\sqrt{4\tan^2\theta}=2\tan\theta$$

$$\boxed{\int \frac{2}{x\sqrt{x^2-4}}\,dx=\int\frac{2\cdot 2\sec\theta\tan\theta}{2\sec\theta\cdot 2\tan\theta}\,d\theta=\int 1\,d\theta=\theta+C=\sec^{-1}\left(\frac{x}{2}\right)+C\ ;\ x>2}$$

This is a pretty cool problem!

14. **Evaluate the integral.** $\int x^4 \cos(2x)\, dx$

Hint: Use the table or chart method.

sign	u'	$\int dv$
$+$	x^4	$\cos(2x)$
$-$	$4x^3$	$\frac{1}{2}\sin(2x)$
$+$	$12x^2$	$-\frac{1}{4}\cos(2x)$
$-$	$24x$	$-\frac{1}{8}\sin(2x)$
$+$	24	$\frac{1}{16}\cos(2x)$
$-$	0	$\frac{1}{32}\sin(2x)$

sign	u'	$\int dv$
$+$	x^4	$\cos(2x)$
$-$	$4x^3$	$\frac{1}{2}\sin(2x)$
$+$	$12x^2$	$-\frac{1}{4}\cos(2x)$
$-$	$24x$	$-\frac{1}{8}\sin(2x)$
$+$	24	$\frac{1}{16}\cos(2x)$
$-$	0	$\frac{1}{32}\sin(2x)$

$$\int x^4 \cos(2x)\, dx = \frac{1}{2}x^4\sin(2x) + x^3\cos(2x) - \frac{3}{2}x^2\sin(2x) - \frac{3}{2}x\cos(2x) + \frac{2}{3}\sin(2x) + C$$

Extra credit.

Solve one of the following: (circle the one you choose)

6 points $\int \sec^3 dx$ or

5 points $\int \sin^{-1} x\, dx$ **Both in book**