

Find the following limits:

1. $\lim_{x \rightarrow 5} (x^2 - 5x) = 5^2 - 5(5) = 25 - 25 = 0$ 1. 0

Direct substitution.

2. $\lim_{x \rightarrow -4} \frac{x^2 - 16}{x + 4} = \lim_{x \rightarrow -4} \frac{(x+4)(x-4)}{x+4} = \lim_{x \rightarrow -4} (x-4) = -4 - 4 = -8$ 2. -8

Can't directly substitute- why?

3. $\lim_{x \rightarrow 2} \frac{|x-2|}{x-2}$ dne since the RHL \neq LHL, i.e. $-1 \neq 1$ 3. dne

$\lim_{x \rightarrow 2^-} \frac{-(x-2)}{x-2} = \lim_{x \rightarrow 2^-} (-1) = -1, \lim_{x \rightarrow 2^+} \frac{x-2}{x-2} = \lim_{x \rightarrow 2^+} (1) = 1$

Or look at graph.

4. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \lim_{x \rightarrow 0} \frac{2 \sin 2x}{2x} = 2 \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = 2(1) = 2$ 4. 2

Can't directly substitute. Use identity.

5. $\lim_{x \rightarrow 4} \frac{\sqrt{x-2}}{x+7} = \frac{\sqrt{4-2}}{4+7} = \frac{\sqrt{2}}{11}$ 5. $\frac{\sqrt{2}}{11}$

Direct substitution.

6. $\lim_{x \rightarrow 1} \sin\left(\frac{\pi}{3}x\right) = \lim_{x \rightarrow 1} \sin\left(\frac{\pi}{3} \cdot 1\right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ 6. $\frac{\sqrt{3}}{2}$

Direct substitution.

7. $\lim_{x \rightarrow \frac{2\pi}{3}} \cos x = \cos \frac{2\pi}{3} = -\frac{1}{2}$ 7. $-\frac{1}{2}$

Direct substitution.

8. $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x \rightarrow 3} \left[\frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} \right] = \lim_{x \rightarrow 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$
 $= \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x \rightarrow 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{2+2} = \frac{1}{4}$

Can not directly evaluate – why??

Note: $\lim_{x \rightarrow a} f(x) \neq f(x)$. One is limit as x approaches a and the other is a function evaluated at x.

They are not the same thing. Otherwise why would we use limits??