Math 181 Test 1 **Name _KEY_____**

You must show all work!

Find the limit (if it exists)

1. $\lim_{x\to 3}(2x+5) = 2(3)+5 = 11$

2. $\lim_{x\to -2}\dfrac{x^2-2x-8}{x+2} = \lim_{x\to -2}\dfrac{(x+2)(x-4)}{x+2} = \lim_{x\to -2}(x-4) = -2-4 = -6$

   Indeterminate:

3. $\lim_{x\to 3^-}\dfrac{|x-3|}{x-3} = \lim_{x\to 3^-}\dfrac{-(x-3)}{x-3} = \lim_{x\to 3^-}(-1) = -1$

   Since $x<3, |x-3| = -(x-3)$.

4. $\lim_{x\to 1^+}\dfrac{4}{x-1} = \infty$

   Graph or investigate:

x	y
1.1	$\dfrac{4}{1.1-1} = \dfrac{4}{.1} = 40$
1.01	$\dfrac{4}{1.01-1} = \dfrac{4}{.01} = 400$
1.001	$\dfrac{4}{1.001-1} = \dfrac{4}{.001} = 4000$

5.

$$\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 3}\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3}\frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{3+1}+2} = \frac{1}{4}$$

Indeterminate:

Use the definition of the derivative; find $f'(x)$.

6. $f(x) = x^2 + 2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 + 2 - (x^2+2)}{h}$$
$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 2 - x^2 - 2}{h} = \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0}(2x+h) = 2x + 0 = 2x$$

7. Find $\frac{dy}{dx}$ given $xy + y^2 = x$

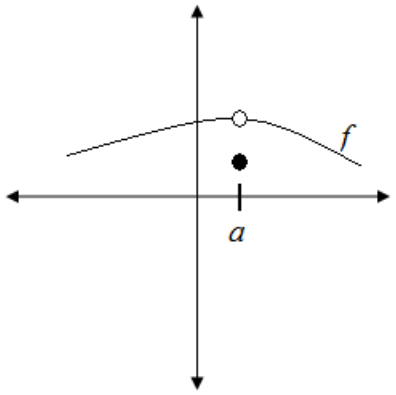
$$\frac{d}{dx}\left[xy + y^2\right] = \frac{d}{dx}[x]$$
$$(1)\, y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 1$$
$$\left(x + 2y\right)\frac{dy}{dx} = 1 - y$$
$$\frac{dy}{dx} = \frac{1-y}{x+2y}$$

Find the x values (if any) at which f is not continuous. State what type of discontinuity.

8. $f(x) = \frac{1}{(x-3)^2}$ when $x = 3$, non-removable

9. $g(x) = \frac{x^2 - 25}{x+5} = \frac{(x-5)(x+5)}{x+5} = x - 5$ when $x = -5$, removable

10. Draw an example of a function, *f*, whose limit exist at value *a* but $\lim_{x\to a} f(x) \neq f(a)$



11. Use the quotient rule to find the derivative:

$$f(\theta) = \csc\theta = \frac{1}{\sin\theta}$$

$$f'(\theta) = \frac{\sin\theta\cdot(0) - 1\cdot\cos\theta}{\sin^2\theta} = \frac{-\cos\theta}{\sin^2\theta} = -\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta} = -\csc\theta\cot\theta$$

Find the derivatives:

12. $y = (2x^2 + 3)^{10}$

$$y' = 10(2x^2+3)^9(4x) = 40x(2x^2+3)^9$$

13. $y = x^3\sin(3x)$

$$y' = 3x^2\sin(3x) + x^3\cos(3x)\cdot 3 = 3x^2\sin(3x) + 3x^3\cos(3x)$$

14. $f(x) = \dfrac{x+2}{x-3}$

$$f'(x) = \frac{(x-3)(1) - (x+2)(1)}{(x-3)^2} = \frac{x-3-x-2}{(x-3)^2} = -\frac{5}{(x-3)^2}$$

15. $g(x) = \tan x$

$g'(x) = \sec^2 x$ just memorize

16. Find $\dfrac{d^2y}{dx^2}$ if $y = \cos x$

$$\frac{dy}{dx} = -\sin x$$

$$\frac{d^2y}{dx^2} = -\cos x$$

Extra credit

Graph and find the limit (if it exists)

$f(x) = \begin{cases} 0 & \text{if } x = n \\ 1 & \text{if } x \neq n \end{cases}$ where n is an integer. FIND $\lim\limits_{x\to n} f(x) = 1$