Math 181 **Test 3** **Name __________________________**

McDonald **Part 1** **Spring 2012**

You must show all work on this test. No Graphing Calculators Ti-89 or Ti-92 or ESP.

#1-2 **Graph** the following. **Identify** all important **features** for each graph.
Name type of test if applicable. FDT, SDT, POI, etc…

10 *points each*

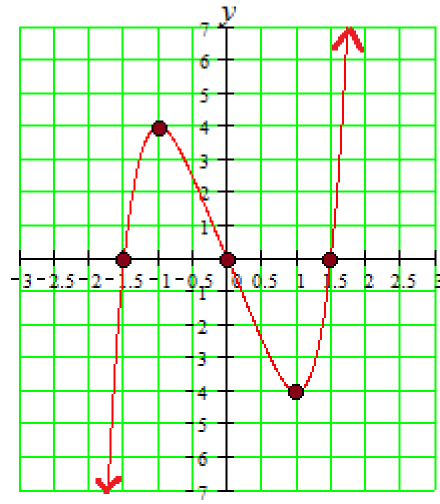
Special features: { Maximums, Minimums, POIs, concavity, **Bonus Pt**: *x*, *y* intercepts}

1. $y = x^5 - 5x$

First, find intercepts. $x^5 - 5x = x(x^4 - 5) = 0,\ x = 0\ \ or\ x^4 - 5 = 0 \Rightarrow x = \pm\sqrt[4]{5}$

NOTE: $\sqrt[4]{5} \approx 1.5$

x-int; $(0,0), (-\sqrt[4]{5}, 0), (\sqrt[4]{5}, 0)$ *y*-int: (0,0)



Next, increasing, decreasing and extrema…

$y' = 5x^4 - 5$

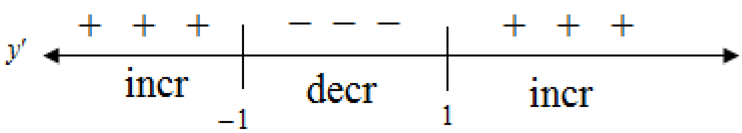
$5x^4 - 5 = 0 \quad note: 5(x^4 - 1) = 5(x^2 - 1)(x^2 + 1)$

$5x^4 = 5$ 2 real, 2 imaginary solutions

$x^4 = 1$ $\therefore$ *Easier way* $\because$.

$x = \pm 1$

Critical Points: $c = -1, 1$



$y'(-2) = 5(-2)^4 - 5 = 75 > 0$

$y'(0) = 5(0)^4 - 5 = -5 < 0$

$y'(2) = 5(2)^4 - 5 = 75 > 0$

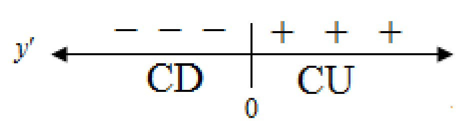
y is increasing on $(-\infty, -1)$ and $(1, \infty)$ and decreasing on $(-1, 1)$.

Therefore, by the **F**irst **D**erivative **T**est, (-1, 4) is a **local maximum**, (1, -4) is **a local minimum**.

Finally, test for Concavity and possible Points Of Inflections…

$y'' = 20x^3,\ 20x^3 = 0 \Rightarrow x = 0.$

Critical Point for Concavity: $k = 0$



$y''(-1) = 20(-1)^3 = -20 < 0$

$y''(1) = 20(1)^3 = 20 > 0$

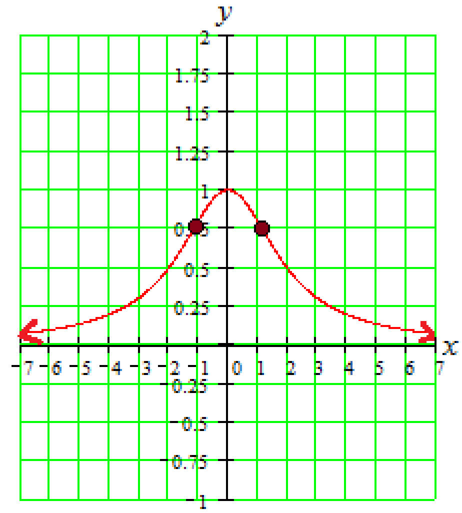
Therefore, (0,0) is a **P**oint **O**f **I**nflection because concavity changes at *x* = 0.

Graph the following. **Identify** all important **features** for each graph.
Name type of test if applicable. FDT, SDT, POI, etc…

10 *points each*

Special features: { Maximums, Minimums, POIs, concavity, **Bonus Pt**: Asymptotes }

2. $f(x) = \dfrac{4}{x^2+4}$



First, find intercepts. $f(0) = \dfrac{4}{0^2+4} = 1$ and $\dfrac{4}{x^2+4} \neq 0$

x-int; None *y*-int: (0,1)

Check for asymptote…HA: $y = 0$

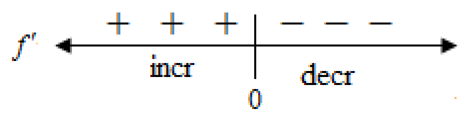
$$\lim_{x \to \pm\infty} \frac{4}{x^2+4} = 0$$

Next, increasing, decreasing and extrema…

$$f(x) = 4\left(x^2+4\right)^{-1}$$

$$f'(x) = (-1)4\left(x^2+4\right)^{-1-1}(2x)$$

$$= -\frac{8x}{(x^2+4)^2},\ c = 0$$



f is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. Therefore, by the **FDT**, (0,1) is a **maximum**.

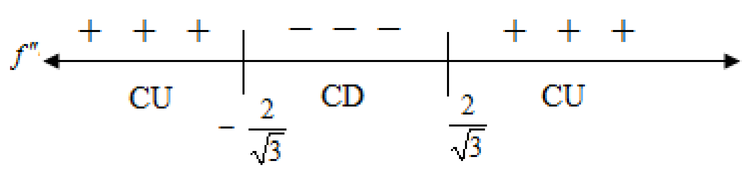
Finally, test for Concavity and possible Points Of Inflections…

$f'(x) = -\dfrac{8x}{(x^2+4)^2}$ Leave in quotient form because we have to set it equal to zero.

$$f''(x) = -8\left[\frac{(x^2+4)^2(1) - x2(x^2+4)(2x)}{\left((x^2+4)^2\right)^2}\right] = -8\left[\frac{(x^2+4)\left[(x^2+4)-4x^2\right]}{(x^2+4)^4}\right] = -\frac{8(4-3x^2)}{(x^2+4)^3} = \frac{8(3x^2-4)}{(x^2+4)^3}$$

Setting $f''(x) = 0$ is equivalent to setting $3x^2 - 4 = 0$. $3x^2 = 4 \Rightarrow x = \pm\dfrac{2}{\sqrt{3}}$

$f''(-2) = \dfrac{1}{8} > 0,\ f''(0) = -\dfrac{1}{2} < 0,\ f''(2) = \dfrac{1}{8} > 0$



It is concave up on $(-\infty, -\frac{2}{\sqrt{3}})$ and $(\frac{2}{\sqrt{3}}, \infty)$ and concave down on $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$.

Therefore $\left(-\dfrac{2}{\sqrt{3}}, \dfrac{3}{4}\right)$ and $\left(\dfrac{2}{\sqrt{3}}, \dfrac{3}{4}\right)$ are POI's. Note the function is even.

You must show all work on this test. No Ti-89 or Ti-92 or ESP.

5 *points each*
Evaluate the following:

3. $\int 5\,dx = 5x + C$

4. $\int 2\cos x\,dx = 2\sin x + C$

5. $\int (x^2 - x + 5)\,dx = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 5x + C$

6. $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} + C = -x^{-1} + C = -\frac{1}{x} + C$

7. $\int \sec^2 y\,dy = \tan y + C$

8. $\int 3\cos\theta\,d\theta = 3\sin\theta + C$

9. $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

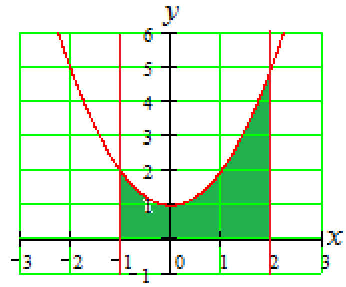
#10-11 Find the **area** of the region bounded by the graphs of the equations. HINT: Draw a picture.

10. $y = x^2 + 1$, $y = 0, x = -1, x = 2$

10. 6 sq units

$$\int_{-1}^{2} (x^2 + 1)\,dx = \frac{1}{3}x^3 + x\Big|_{-1}^{2} = \frac{1}{3}(2)^3 + 2 - \left(\frac{1}{3}(-1)^3 + (-1)\right)$$

$$= \frac{8}{3} + 2 - \left(-\frac{1}{3} - 1\right) = \frac{8}{3} + 2 + \frac{1}{3} + 1 = \frac{9}{3} + 3 = 6$$



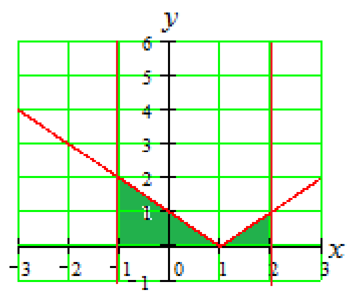
11. $y = |x - 1|$, $y = 0, x = -1, x = 2$

11. $\frac{5}{2}$ sq units

$$\int_{-1}^{1} -(x-1)\,dx + \int_{1}^{2} (x-1)\,dx = \left[-\frac{1}{2}x^2 + x\right]_{-1}^{1} + \left[\frac{1}{2}x^2 - x\right]_{1}^{2}$$

$$= -\frac{1}{2}(1)^2 + 1 - \left(-\frac{1}{2}(-1)^2 + (-1)\right) + \frac{1}{2}2^2 - 2 - \left(\frac{1}{2}(1)^2 - (1)\right)$$

$$= -\frac{1}{2} + 1 + \frac{1}{2} + 1 + 2 - 2 - \frac{1}{2} + 1 = 3 - \frac{1}{2} = \frac{5}{2}$$



Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

12. $F(x) = \int_a^x (t + 4)\,dt = x + 4$

13. $F(x) = \int_a^{x^3} t \sin t\,dt = x^3 \sin x^3 (3x^2) = 3x^5 \sin x^3$

Evaluate the definite and indefinite integrals.

14. $\int_0^1 2x(x^2+3)^5\,dx$ 14. $\frac{3367}{6}$

$\int_0^1 2x(x^2+3)^5\,dx$ Let $u=x^2+3$ $x=0, u=0^2+3=3$

$du=2x\,dx$ $x=1, u=1^2+3=4$

$$=\int_3^4 u^5\,du=\frac{1}{6}u^6\Big|_3^4=\frac{1}{6}\left[4^6-3^6\right]=\frac{3367}{6}$$

15. $\int x^3\cos x^4\,dx$ 15. $\frac{1}{4}\sin x^4+C$

Let $u=x^4$

$\int x^3\cos x^4\,dx$ $du=4x^3\,dx$

$\frac{1}{4}du=x^3\,dx$

$$=\frac{1}{4}\int\cos u\,du=\frac{1}{4}\sin u+C=\frac{1}{4}\sin x^4+C$$

16. $\int\frac{2x}{\sqrt{4-x^2}}\,dx$ 16. $-2\sqrt{4-x^2}+C$

$\int\frac{2x}{\sqrt{4-x^2}}\,dx$ Let $u=4-x^2$

$du=-2x\,dx$

$$=-\int\frac{1}{u^{\frac{1}{2}}}\,du=-\int u^{-\frac{1}{2}}\,du=-\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C=-\frac{(4-x^2)^{\frac{1}{2}}}{\frac{1}{2}}+C=-2\sqrt{4-x^2}+C$$

Extra Credit: Pick only one of these for extra credit. *Use Back of Test.*

4 *points*
Evaluate: $\int x\sqrt{4x+1}\,dx$

7 *points*
Graph: $y=x\sqrt{4-x}$. List all features.

You are finished. Math is goooood!!!!!