Math 181 **Test 2** **Name** ______________________

McDonald **Spring 2012**

You must show all work on this test. No Ti-89 or Ti-92 or ESP.

5 *points each*

Find the derivative of the following functions.

1. $y=\left(2x^3-7\right)^{10}$

1. $\boxed{y'=60x^2\left(2x^3-7\right)^9}$

$$y'=10\left(2x^3-7\right)^{10-1}(2x^3-7)'=10\left(2x^3-7\right)^{10-1}6x^2=60x^2\left(2x^3-7\right)^9$$

Power Rule w/Chain Rule

$y=u^n$, where $u=g(x)$

$y'=nu^{n-1}\cdot u'$

2. $f(x)=\cos(5x)$

2. $\boxed{f'(x)=-5\sin(5x)}$

$$f'(x)=-\sin(5x)(5)=-5\sin(5x)$$

Don't forget the chain rule. The argument (or angle) of sine, 5*x*, is not affected. You still have to take the derivative of 5*x* which is 5.

3. $h(\theta)=\tan^2(3\theta)=\left[\tan(3\theta)\right]^2$

3. $\boxed{h'(\theta)=6\tan(3\theta)\sec^2(3\theta)}$

$$h'(\theta)=2\left[\tan(3\theta)\right]^{2-1}\sec^2(3\theta)(3)=6\tan(3\theta)\sec^2(3\theta)$$

Note: $\frac{d}{dx}\left[\tan u\right]=\sec^2 u\cdot u'=u'\sec^2 u$ The angle does not change when taking the derivative of a trig function.

4. $y=\dfrac{x}{\sqrt{x^4+4}}=\dfrac{x}{(x^4+4)^{\frac{1}{2}}}$

4. $\boxed{y'=\dfrac{4-x^4}{(x^4+4)^{\frac{3}{2}}}}$

$$y'=\frac{(x^4+4)^{\frac{1}{2}}(1)-x(\frac{1}{2})(x^4+4)^{-\frac{1}{2}}(4x^3)}{\left[(x^4+4)^{\frac{1}{2}}\right]^2}=\frac{(x^4+4)^{\frac{1}{2}}-2x^4(x^4+4)^{-\frac{1}{2}}}{(x^4+4)^1}$$

$$=\frac{(x^4+4)^{-\frac{1}{2}}\left[(x^4+4)^1-2x^4\right]}{(x^4+4)^1}=\frac{4-x^4}{(x^4+4)^{\frac{3}{2}}}$$

5 *points*

Find the $\frac{dy}{dx}$ by implicit differentiation.

5. $5x^2 - 4y^3 = 9$

5. $\boxed{\frac{dy}{dx} = \frac{5x}{6y^2}}$

$$\frac{d}{dx}\left[5x^2 - 4y^3\right] = \frac{d}{dx}[9]$$
$$10x - 12y^2\frac{dy}{dx} = 0$$
$$-12y^2\frac{dy}{dx} = -10x$$
$$\frac{dy}{dx} = \frac{-10x}{-12y^2} = \frac{5x}{6y^2}$$

5 *points*

Find the $\frac{dy}{dx}$ by implicit differentiation.

6. $x^2y + xy^3 = 2x$

6. $\boxed{\frac{dy}{dx} = \frac{2 - 2xy - y^3}{x^2 + 3xy^2}}$

$$\frac{d}{dx}\left[x^2y + xy^3\right] = \frac{d}{dx}[2x]$$
$$2xy + x^2\frac{dy}{dx} + (1)y^3 + x(3y^2)\frac{dy}{dx} = 2$$
$$x^2\frac{dy}{dx} + 3xy^2\frac{dy}{dx} = 2 - 2xy - y^3$$
$$\left[x^2 + 3xy^2\right]\frac{dy}{dx} = 2 - 2xy - y^3$$
$$\frac{dy}{dx} = \frac{2 - 2xy - y^3}{x^2 + 3xy^2}$$

10 points

7. Find **all extrema** on the **closed** interval [0, 4] and classify the extrema using the first derivative test.

$f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$

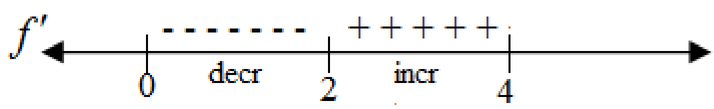
$3x^2 - 12 = 3(x^2 - 4) = 0$, $x^2 - 4 = 0$

$x^2 = 4 \Rightarrow x = \pm 2$

The critical values are $c = -2, 2$. Ignore -2 because it is not in [0, 4].

Test points: $f'(x) = 3(1)^2 - 12 = -9 < 0$

$f'(x) = 3(3)^2 - 12 = 15 > 0$



$f(0) = 0,\ f(2) = 2^3 - 12(2) = -16$, and $f(4) = 4^3 - 12(4) = 16$

By the FDT, $(2,-16)$ is a global minimum, $(4,16)$ is a global maximum, and $(0,0)$ is a relative maximum.

You may also do this using the definition extrema on a closed interval.

10 points

8. Find **the intervals** where the function is decreasing, increasing or constant.

$f(x) = x^4 - 4x^3 - 8x^2$

Find critical points: $f'(x) = 4x^3 - 12x^2 - 16x$, $4x(x^2 - 3x - 4) = 4x(x-4)(x+1)$

$x = 0, x - 4 = 0,\ x + 1 = 0$ gives $c = -1, 0, 4$

interval	$(-\infty,-1)$	$(-1,0)$	$(0,4)$	$(4,\infty)$
$f'(x)$	$f'(-2) = -48 < 0$	$f'(-\frac{1}{2}) = \frac{9}{2} > 0$	$f'(-2) = -24 < 0$	$f'(5) = 120 > 0$
conclusion	decreasing	increasing	decreasing	increasing

Or use line graph method.

10 points

9. Let $f''(x) = 4x^3 - 2x$ and let $f(x)$ have critical numbers -2, 0, 1. Use the **second derivative** test to determine if any of the critical numbers gives a local minimum or maximum.

$f''(-2) = 4(-2)^3 - 2(-2) = -8 < 0$ By SDT, (local) maximum. Concave Down

$f''(0) = 4(0)^3 - 2(0) = 0$ By SDT, no conclusion.

$f''(1) = 4(1)^3 - 2(1) = 2 > 0$ By SDT, (local) minimum. Concave Up

This is the Second Derivative Test.

10 *points*

11. Find **the open intervals** on which the graph is **concave** up or **concave** down.

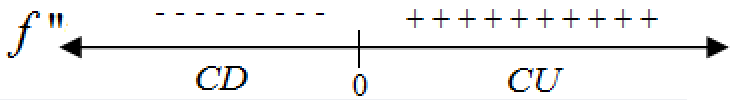
$f(x) = x^5 - 5x$

We need to find the second derivative so we can test for concavity.

$f'(x) = 5x^4 - 5$

$f''(x) = 20x^3$

So possible POI is $k = 0$.



Therefore, f is concave up down on $(-\infty, 0)$ and concave up on $(0, \infty)$.

10 *points*

12. Find the all extrema for $f(x) = 2x\sqrt{x+3}$ using any method.

$$f'(x) = 2\left[(1)(x+3)^{\frac{1}{2}} + x\left(\tfrac{1}{2}\right)(x+3)^{\frac{1}{2}-1}(1)\right] = 2(x+3)^{\frac{1}{2}} + x(x+3)^{-\frac{1}{2}}$$

$$= (x+3)^{-\frac{1}{2}}\left[2(x+3)^1 + x\right] = \frac{3x+6}{(x+3)^{\frac{1}{2}}}$$

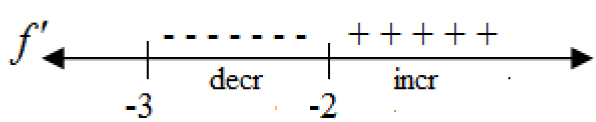
$$\frac{3x+6}{(x+3)^{\frac{1}{2}}} = 0 \Rightarrow 3x+6 = 0 \Rightarrow x = -2$$

Note: $f'(-3) = undefined$

$$f'(-2.5) = \frac{3(-2.5)+6}{(-2.5+3)^{\frac{1}{2}}} = \frac{-1.5}{\sqrt{0.5}} < 0$$

$c = -2, -3$ are critical numbers.

$$f'(0) = \frac{3(0)+6}{(0+3)^{\frac{1}{2}}} = \frac{6}{\sqrt{3}} > 0$$



$f(-3) = 2(-3)\sqrt{-3+3} = -6(0) = 0$, $f(-2) = 2(-2)\sqrt{-2+3} = -4(1) = -4$

By the FDT, (–3, 0) is a local maximum and (–2, –4) is a minimum (global).

10 points

For #13 $f(x) = 2x^3 - 3x^2$

a) Find all critical numbers of *f*.

$$f'(x) = 6x^2 - 6x$$
$$6x(x-1) = 0 \quad x = 0 \text{ or } x = 1$$

$c = 0,1$

b) Locate local extrema using the **second derivative test**.

$f''(x) = 12x - 6$, Therefore, by the SDT

$f''(0) = 12(0) - 6 = -6 < 0$ $(0,0)$ is a maximum (local)

$f''(1) = 12(1) - 6 = 6 > 0$ $(1,-1)$ is a minimum (local)

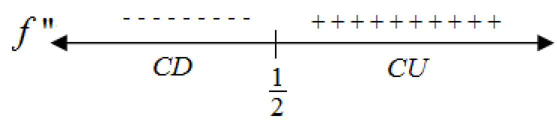
Note: $f(0) = 2(0)^3 - 3(0)^2 = 0$ and $f(1) = 2(1)^3 - 3(1)^2 = -1$

3 Bonus Points

Determine points of inflection using concavity.

$$f''(x) = 12x - 6 = 6(2x-1) = 0,\ 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$$

$k = \frac{1}{2}$, a possible point of inflection.



Therefore, $\left(\frac{1}{2}, -\frac{1}{2}\right)$ is a Point Of Inflection.

$$f\left(\tfrac{1}{2}\right) = 2\left(\tfrac{1}{2}\right)^3 - 3\left(\tfrac{1}{2}\right)^2 = \frac{2}{8} - \frac{3}{4} = -\frac{1}{2}$$

10 *points*

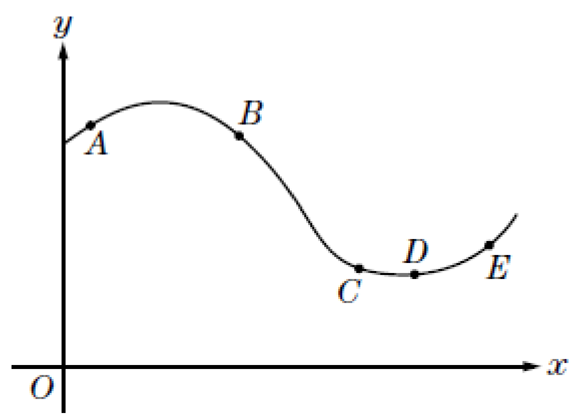
At which of the five points on the graph in the figure at the right is …

14. $\frac{dy}{dx} > 0$ Increasing. A & E

15. $\frac{d^2y}{dx^2} < 0$ Concave Down: A & B

2 Bonus points

$\frac{dy}{dx} = 0$ Max or Min: D



Extra Credit

A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

(a) 25 cm ?

$$800 = 4\pi(25)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{800}{4\pi(25)^2} = \frac{8}{25\pi}\, cm/\min$$

(b) 50 cm ?

$$800 = 4\pi(50)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{800}{4\pi(50)^2} = \frac{2}{25\pi}\, cm/\min$$

You are finished. Yeah you!!!!!