Math 181 **Test 1** **Name** **Solutions**

You must show all work! And simplify, when possible. Spring 2012

5 points each unless noted.

Find the limit, if exists. Write infinity if the limit is infinite.

1. $\lim_{x\to 2}(5x+3)=5(2)+3=13$ — 1. 13

2. Numerator and denominator have same degree. — 2. $\frac{5}{4}$

$$\lim_{x\to\infty}\frac{5x^2-4x+1}{4x^2+1}=\lim_{x\to\infty}\frac{\frac{5x^2-4x+1}{x^2}}{\frac{4x^2+1}{x^2}}=\lim_{x\to\infty}\frac{\frac{5x^2}{x^2}-\frac{4x}{x^2}+\frac{1}{x^2}}{\frac{4x^2}{x^2}+\frac{1}{x^2}}$$

$$=\lim_{x\to\infty}\frac{5-\frac{4}{x}+\frac{1}{x^2}}{4+\frac{1}{x^2}}=\frac{5-0+0}{4+0}=\frac{5}{4}$$

3. $\lim_{x\to\infty}\frac{3}{x+1}=0$ Think: $\frac{3}{100,000}=.00003$ — 3. 0

The degree in the numerator is less than the degree in the denominator.

4. $\lim_{x\to 0}\frac{5\sin x}{x}=5\lim_{x\to 0}\frac{\sin x}{x}=5(1)=5$ — 4. 5

Memorize!!

5. $\lim_{x\to 3}(x^2-2x+1)=3^2-2(3)+1=9-6+1=4$ — 5. 4

6. $\lim_{x\to 5}\frac{x^2-25}{x-5}=\lim_{x\to 5}\frac{(x-5)(x+5)}{x-5}=\lim_{x\to 5}(x+5)=5+5=10$ — 6. 10

Removable discontinuity. i.e. *a hole….*

7. $\lim_{x\to 3}9=9$ — 7. 9

8. $\lim_{x\to 1^+}\frac{4}{x-1}=\infty$ $\frac{1}{1.0001-1}=\frac{1}{.0001}=10,000$ — 8. DNE, ∞

Try a number close to 1 coming from the right side.

Find the **derivative** of each function.

9. $h(x) = 5x^2 - 3x + 1$

$h'(x) = 2(5)x^{2-1} - 3x^{1-1} = 10x - 3$

9. $h'(x) = 10x - 3$

10. $y = \dfrac{3}{x^3} = 3x^{-3}$

$\dfrac{dy}{dx} = -3(3)x^{-3-1} = -9x^{-4} = -\dfrac{9}{x^4}$

10. $\dfrac{dy}{dx} = -\dfrac{9}{x^4}$

Note:

i) You may use either y' *or* $\dfrac{dy}{dx}$ for the derivative of $y = f(x)$.

ii) $\dfrac{d}{dx}[h(x)]$ *or* $\dfrac{dh}{dx}$ may be used for $h'(x)$.

11. $f(x) = \sqrt{x} + 2x^{\frac{3}{2}} = x^{\frac{1}{2}} + 2x^{\frac{3}{2}}$

$f'(x) = \dfrac{1}{2}x^{\frac{1}{2}-1} + \dfrac{3}{2}\cdot 2x^{\frac{3}{2}-1} = \dfrac{1}{2}x^{-\frac{1}{2}} + 3x^{\frac{1}{2}} = \dfrac{1}{2x^{\frac{1}{2}}} + 3x^{\frac{1}{2}}$

$= \dfrac{1}{2\sqrt{x}} + 3\sqrt{x}$

11. $\dfrac{1}{2\sqrt{x}} + 3\sqrt{x}$

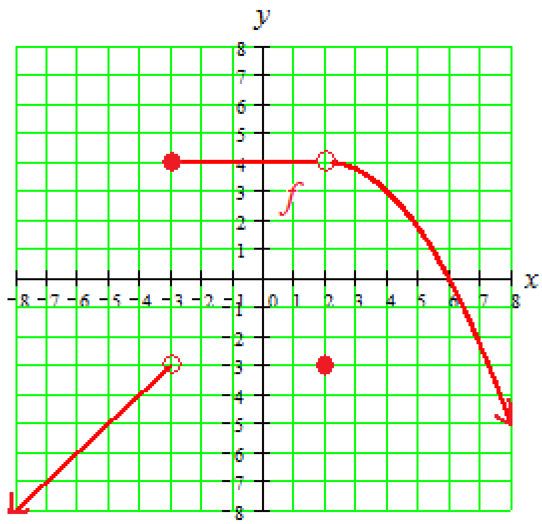
12. $f(x) = \dfrac{x^3 - 2}{x^2} = \dfrac{x^3}{x^2} - \dfrac{2}{x^2} = x - 2x^{-2}$

$f'(x) = 1 - (-2)2x^{-2-1} = 1 + 4x^{-3} = 1 + \dfrac{4}{x^3}$

$(0r)\dfrac{x^3 + 4}{x^3}$

12. $f'(x) = 1 + \dfrac{4}{x^3}$

Use Graph Below. Approximate to nearest integer, if necessary.



13. $f(2) =$ -3

14. $\lim\limits_{x \to 2} f(x) =$ 4 LH limit = RH Limit

Removable discontinuity

15. $f(-3) =$ 4

16. $\lim\limits_{x \to -3} f(x) = \text{DNE}$

$\lim\limits_{x \to -3^-} f(x) = -3 \neq 4 = \lim\limits_{x \to -3^+} f(x)$

10 points

17. Use the **definition** a derivative to find $f'(x)$ where $f(x) = 2x - 1$.

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 1 - (2x-1)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x + 2(\Delta x) - 1 - 2x + 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} 2 = 2$$

Don't skip steps.

10 points

18. Find **the slope** of the tangent line to the graph of the function at the given point.

$f(x) = x^2$ at $(-3, 9)$

$f'(x) = 2x$

$m = f'(-3) = 2(-3) = -6$

The slope of the tangent line is $m = -6$.

The Equation of the Tangent Line.

$y - y_1 = m(x - x_1)$

$y - 9 = -6(x - (-3))$

$y - 9 = -6x - 18$

$y = -6x - 9$

Extra Credit: Find the derivative using the **limiting process**. i.e. $\delta - \varepsilon$ *proof* definition

Let $f(x) = 5x + 6$, prove $\lim_{x \to 2} f(x) = 16$.

$\forall \varepsilon > 0, \exists \delta > 0$ such that $|x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$. Hint: Find a delta that makes it true,

Given: $|x - 2| < \delta$,

$|5x + 6 - (16)| = |5x - 10| = 5|x - 2| < 5\delta = \varepsilon$

$\therefore$ Let $\delta = \frac{\varepsilon}{5}$.