Math 182 Test 2 Name Key Master

McDonald Date 10/14/2004

7 Points each

1. Find the derivative of $f(x) = 5^{x^2-3x}$.
Chain rule

$\boxed{f'(x) = 5^{x^2-3x}(\ln 5)(2x-3)}$ or $(2x\ln 5 - 3\ln 5)5^{x^2-3x}$

2. Find the derivative of $y = x^2 e^{-x}$.
Product rule

$y' = 2xe^{-x} + x^2(-1)e^{-x} = 2xe^{-x} - x^2e^{-x}$ or $\boxed{(2x - x^2)e^{-x}}$

or $\dfrac{(2x - x^2)}{e^x}$

3. Use implicit differentiation to find $\dfrac{dy}{dx}$.

$3e^x + 2y - \ln\dfrac{x}{y} = 3$

Simplify first…

$3e^x + 2y - (\ln x - \ln y) = 3$

$3e^x + 2y - \ln x + \ln y = 3$

$3e^x + 2y' - \dfrac{1}{x} + \dfrac{y'}{y} = 0 \quad 2y' + \dfrac{y'}{y} = \dfrac{1}{x} - 3e^x$

$\left(2 + \dfrac{1}{y}\right)y' = \dfrac{1}{x} - 3e^x \qquad y' = \dfrac{\frac{1}{x} - 3e^x}{2 + \frac{1}{y}}$

Now simplify...yes, even if I don't tell you.

$$y' = \frac{\left(\frac{1}{x} - 3e^x\right)\cdot xy}{\left(2 + \frac{1}{y}\right)\cdot xy} = \boxed{\frac{y - 3xye^x}{2xy + x}}$$

4. Show if the function has an inverse and find the inverse. (if possible)
State the domain and *range* of each function.

$g(x) = x^2 - 4$ for $-\infty < x < \infty$ or $x \in \mathbb{R}$.

$g'(x) = 2x > 0$ for $x > 0$ therefore g is strictly increasing and is one-to-one and has an inverse

$g'(x) = 2x < 0$ for $x < 0$ therefore g is strictly decreasing and is one-to-one and has an inverse

BUT, $g(x)$ is neither strictly increasing or decreasing throughout its entire domain so it has **no** inverse (not one to one)

Domain of g: $-\infty < x < \infty$ or $x \in \mathbb{R}$

Range of g: $[-4, \infty)$

5. Find the derivative of $y=(x+2)^{10}\sqrt{x-6}$ using logarithmic differentiation.

$$\ln y=\ln\left[(x+2)^{10}\sqrt{x-6}\right]$$

$$\ln y=\ln(x+2)^{10}+\ln\left(x-6\right)^{\frac{1}{2}}$$

$$\ln y=10\ln(x+2)+\tfrac{1}{2}\ln\left(x-6\right)$$

$$\frac{y'}{y}=\frac{10}{x+2}+\frac{1}{2(x-6)}$$

$$y'=y\left[\frac{10}{x+2}+\frac{1}{2(x-6)}\right]=\boxed{(x+2)^{10}\sqrt{x-6}\left[\frac{10}{x+2}+\frac{1}{2(x-6)}\right]}$$

6. Evaluate $\int \frac{5^x}{5^x-6}\,dx$

Let $u=5^x-6$

$du=5^x\ln 5\,dx$

$\frac{1}{\ln 5}du=5^x\,dx$

$$\frac{1}{\ln 5}\int\frac{1}{u}\,du=\frac{1}{\ln 5}\ln\left|5^x-6\right|+c=\boxed{\frac{\ln\left|5^x-6\right|}{\ln 5}+c}$$

7. Find $f'(x)$ if $f(x)=\sin^{-1}(3x)$

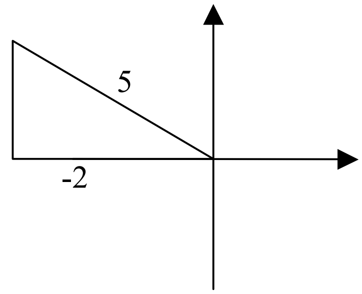
$$f'(x)=\frac{1}{\sqrt{1-(3x)^2}}(3)=\boxed{\frac{3}{\sqrt{1-9x^2}}}$$

8. Evaluate $\int\frac{3}{x\sqrt{x^2-9}}\,dx=3\left(\frac{1}{3}\right)\sec^{-1}\frac{x}{3}+c=\boxed{\sec^{-1}\frac{x}{3}+c}$ $a=3$

9. Evaluate $\int\frac{1}{x^2+2}\,dx=\boxed{\frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}+c}$ $a=\sqrt{2}$

10. Evaluate $\tan\left[\cos^{-1}\left(-\frac{2}{5}\right)\right]$ exactly.

$\sqrt{5^2-2^2}=\sqrt{21}$



$\cos^{-1}\left(-\frac{2}{5}\right)=\theta$

$\cos\theta=-\frac{2}{5}=\frac{adj}{hyp}$ in Quadrant II

$$\tan\theta=\frac{opp}{adj}=\frac{\sqrt{21}}{-2}=-\frac{\sqrt{21}}{2}$$

11. Evaluate $\int \frac{\ln x}{x}\,dx$

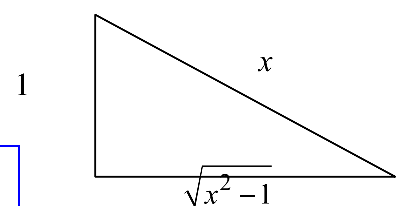
Let $u=\ln x$

$du=\frac{1}{x}dx$

$$\int \frac{\ln x}{x}\,dx=\int \ln x\cdot\tfrac{1}{x}\,dx=\int u\,du=\tfrac{1}{2}u^2+c=\tfrac{1}{2}(\ln x)^2+c$$

12. Rewrite the algebraic expression in x for $x>0$.

$\cos\left(\sin^{-1}\left(\frac{1}{x}\right)\right)$



$\sin^{-1}\left(\frac{1}{x}\right)=\theta$

$\sin\theta=\frac{1}{x}=\frac{opp}{hyp}$

$$\cos\theta=\frac{adj}{hyp}=\frac{\sqrt{x^2-1}}{x}$$

13. List the domain and range of $y=\cos^{-1}x$.

Domain: $[-1,1]$
Range: $[0,\pi]$

14. Evaluate $\int_1^2 e^x\,dx=e^x\Big]_1^2=e^2-e$