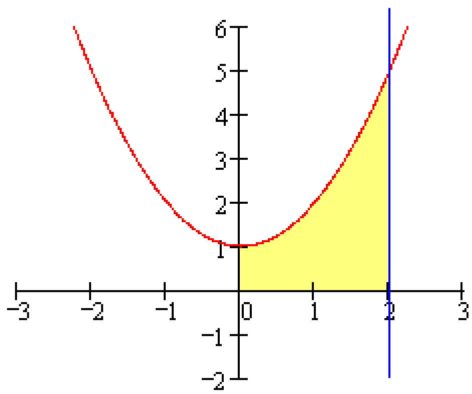
Math 182 Test 1 *Print Name* ____________________

McDonald – Fall 2004

You must show all work! 8 points each

1. Find the **area** of the region bounded by the graphs: Please Graph!

$y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 2$

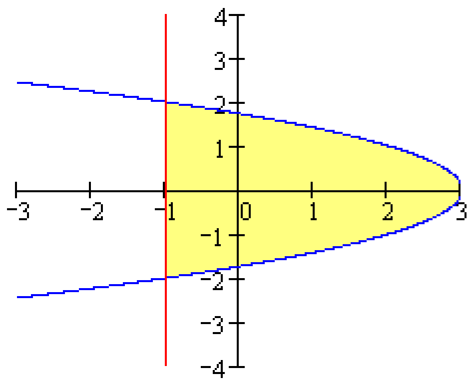


$$\int_0^2 \left(x^2+1\right) dx = \frac{x^3}{3} + x\Big|_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$$

$$f(x) - g(x) = \left(x^2+1\right) - 0 = x^2 + 1$$

2. Find the **area** of the region bounded by the graphs: Please Graph!

$x = 3 - y^2$ and $x = -1$

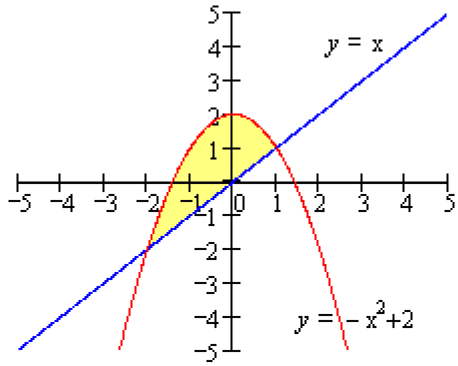


$$\int_{-2}^{2} (4 - y^2)\, dy = 2\int_0^2 (4 - y^2)\, dy = \frac{32}{3}$$

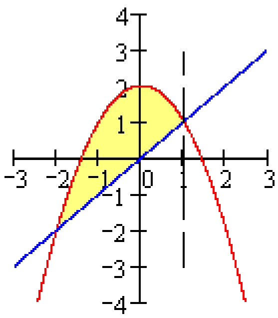
$$f(y) - g(y) = \left(3 - y^2\right) - (-1) = 4 - y^2$$

#3-5 **SET UP ONLY**

Set up an integral that can be used to find the **volume** of the **solid** obtained by revolving the shaded region about the indicated axis. Show work for limits.



3. $x = 1$



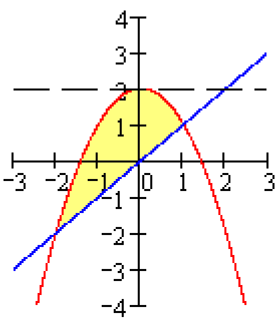
Shell Method

$$2\pi\int_{-2}^{1}(1-x)(-x^2-x+2)\,dx$$

Washer Method

$$\pi\int_{-2}^{1}\left(1+\sqrt{2-y}\right)^2-(1-y)^2\,dy+\pi\int_{1}^{2}\left(1+\sqrt{2-y}\right)^2-\left(1-\sqrt{2-y}\right)^2\,dy$$

4. $y = 2$



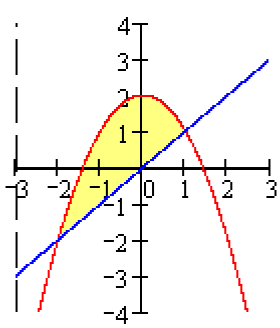
Washer Method

$$\pi\int_{-2}^{1}(2-x)^2-(x^2)^2\,dx$$

Shell Method

$$2\pi\int_{-2}^{1}(2-y)\left(y+\sqrt{2-y}\right)\,dy+2\pi\int_{1}^{2}(2-y)\left(2\sqrt{2-y}\right)\,dy+$$

5. $x = -3$



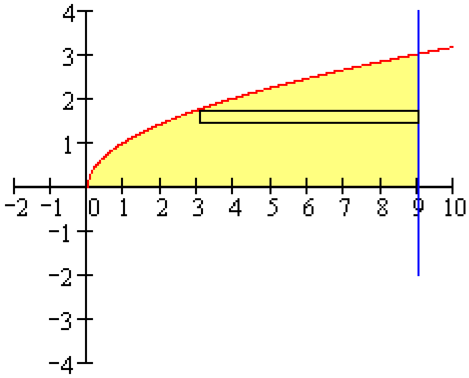
Shell Method

$$2\pi\int_{-2}^{1}(x-(-3))(-x^2-x+2)\,dx$$

Washer Method

$$\pi\int_{-2}^{1}(3+y)^2-\left(3-\sqrt{2-y}\right)^2\,dy+\pi\int_{1}^{2}\left(3+\sqrt{2-y}\right)^2-\left(3-\sqrt{2-y}\right)^2\,dy$$

6. Find the **volume** of the solid formed by revolving the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$ about the line y-axis. Use **Washer Method**.

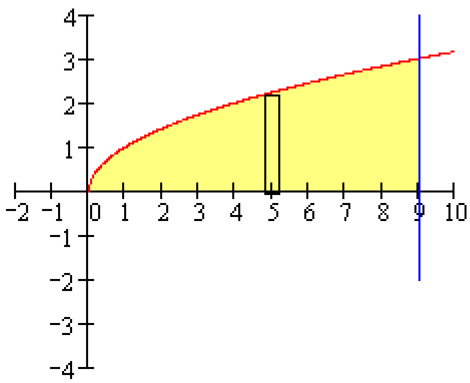


$R = 9$

$r = y^2$

$$\pi \int_0^3 9^2 - (y^2)^2 \; dy = \pi \int_0^3 81 - y^4 \; dy = \frac{972\pi}{5} \text{ units}^3$$

7. Find the **volume** of the solid formed by revolving the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$ about the line y-axis. Use **Shell Method.**



$radius = x$

$height = \sqrt{x}$

$width = dx$

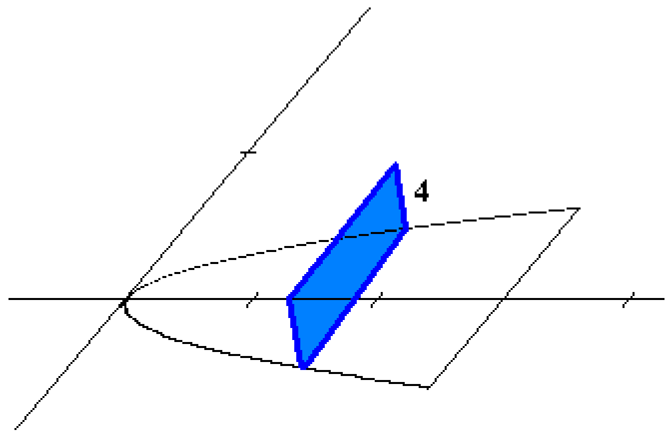
$$2\pi \int_0^9 x\sqrt{x} \; dx = 2\pi \int_0^9 x^{\frac{3}{2}} \; dx = \frac{972\pi}{5} \text{ units}^3$$

8. Let R be the region bounded by the graphs of $x = y^2$ and $x = 9$. Find the **volume** of the **solid** that has R as its base if every cross section by a plane perpendicular to the x-axis has the shape of a rectangle of height 4.

$x = y^2$

$x = 9$

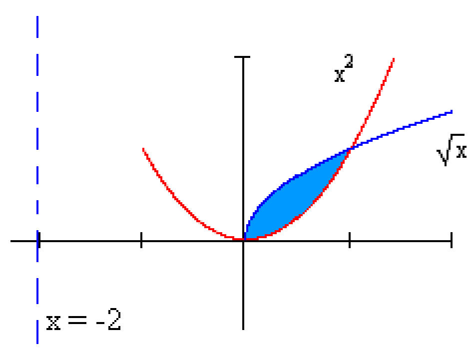
Rectangle of height 4



$$\int_0^9 A(x)\ dx = \int_0^9 4(2\sqrt{x})\ dx = \int_0^9 8x^{\frac{1}{2}} dx = 144\ \ unit^3$$

9. **SET UP ONLY**

Use **Any Method** to find the **volume** of the **solid** formed by revolving the region bounded about the line $\boldsymbol{x} = \mathbf{-2}$.



Shell Method

$$2\pi\int_0^1 (x-(-2))(\sqrt{x}-x^2)\ dx = 2\pi\int_0^1 (x+2)(\sqrt{x}-x^2)\ dx$$

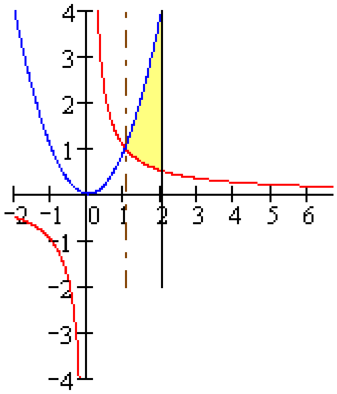
Washer Method

$$\pi\int_0^1 (\sqrt{y}-(-2))^2-(y^2-(-2))^2\ dx = \pi\int_0^1 (\sqrt{y}+2)^2-(y^2+2)^2\ dx$$

$$=\frac{49\pi}{30}units^3$$

10. **Set up** an integral that can be used to find the **volume** of the **solid** formed by revolving the region bounded by

$y = \frac{1}{x}$, $y = x^2$, and $x = 2$ about the line $x = 1$. Any method.



Shell Method

$width = dx$

$radius = x - 1$

$height = x^2 - \frac{1}{x}$

$$2\pi \int_1^2 (x-1)\left(x^2 - \frac{1}{x}\right) dx$$

Washer Method – 2 parts the hard way

$$\pi \int_{\frac{1}{2}}^{1} \left(1^2 - \left(\frac{1}{y} - 1\right)^2\right) dy + \pi \int_1^4 \left(1^2 - \left(\sqrt{y} - 1\right)^2\right) dy$$

11. Find the **arc length** of the graph of the equation from [4, 11] on the x axis.

$f(x) = \frac{2}{3}(x+5)^{\frac{3}{2}}$

$$f'(x) = \frac{2}{3} \cdot \frac{3}{2}(x+5)^{\frac{3}{2}-1} = (x+5)^{\frac{1}{2}}$$

$$\int_4^{11} \sqrt{1 + \left[\sqrt{x+5}\right]^2}\, dx = \int_4^{11} \sqrt{1 + x + 5}\, dx = \int_4^{11} (x+6)^{\frac{1}{2}} dx \approx 25.6$$

12. Write the definite integral that represents the **arc length** of one period of the curve $y = \sin 2x$ (Do not solve) You can't anyway yet!

$y' = 2\cos 2x$

$$\int_0^{\pi} \sqrt{1+[2\cos 2x]^2}\,dx = \int_0^{\pi} \sqrt{1+4\cos^2 2x}\;dx$$

Extra Credit:

Find the **arc length** of the graph of the equation from (–8, 6) to $(-1, \frac{3}{2})$.

$$f(x) = \tfrac{3}{2}x^{\frac{2}{3}}$$

$5\sqrt{5} - 2\sqrt{2}$