14. Use this silly formula $\int \sin au \cos bu \ du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

Evaluate $\int \sin 5x \cos 4x \ dx$ Here $a = 5$ and $b = 4$

$$\int \sin 5x \cos 4x \ du = -\frac{\cos(5-4)x}{2(5-4)} - \frac{\cos(5+4)x}{2(5+4)} + C = \boxed{-\frac{\cos x}{2} - \frac{\cos 9x}{18} + C}$$

Extra Credit: No Stinking Calculators Either!!! PICK ONE. ONLY ONE!!!

Evaluate $\int \frac{1}{x(\sqrt{x} + \sqrt[4]{x})} dx$

$$\int \frac{4u^3 du}{u^4\left(\sqrt{u^4} + \sqrt[4]{u^4}\right)} = 4\int \frac{u^3 du}{u^4\left(u^2 + u\right)} = 4\int \frac{du}{u\left(u^2 + u\right)} = 4\int \frac{du}{u^2(u+1)}$$

Let $u = x^{\frac{1}{4}}$, $u^4 = x$, $4u^3 du = dx$

Partial Fraction

$$4\int \frac{1}{u^2(u+1)} du = 4\int\left[-\frac{1}{u} + \frac{1}{u^2} + \frac{1}{u+1}\right] du = \boxed{4\left(-\ln\left|x^{\frac{1}{4}}\right| - \frac{1}{x^{\frac{1}{4}}} + \ln\left|x^{\frac{1}{4}} + 1\right|\right) + C = 4\left(-x^{-\frac{1}{4}} + \ln\left|\frac{x^{\frac{1}{4}} + 1}{x^{\frac{1}{4}}}\right|\right) + C}$$

Evaluate $\int x\sqrt{5-3x} \ dx$

$$\int \frac{u-5}{-3}\sqrt{u}\left(-\frac{1}{3}\right) du = \frac{1}{9}\int (u-5)u^{\frac{1}{2}} du = \frac{1}{9}\int (u^{\frac{3}{2}} - 5u^{\frac{1}{2}}) du$$

Let $u = 5-3x$, $\frac{u-5}{-3} = x$, $-\frac{1}{3} du = dx$

$$= \frac{1}{9}\cdot\frac{2}{5}u^{\frac{5}{2}} - \frac{5}{9}\cdot\frac{2}{3}u^{\frac{3}{2}} + C = \boxed{\frac{2}{45}(5-3x)^{\frac{5}{2}} - \frac{10}{27}(5-3x)^{\frac{3}{2}} + C}$$