Find the limit if it exists and state the form of indeterminate for each part.

10. $\lim\limits_{x\to 0^+} x^2 \ln x$

$$\lim_{x\to 0^+} x^2 \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}} = \lim_{x\to 0^+} \frac{-x^3}{2x} = \lim_{x\to 0^+} -\frac{x^2}{2} = -\frac{0^2}{2} = \boxed{0}$$

Indeterminate form $0\cdot\infty$, but now $\frac{\infty}{\infty}$. Use L'Hôpital's rule once

11. Evaluate $\int \sin^2 4x\, dx$

Remark $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

Let $\theta = 4x$

$$\sin^2 4x = \frac{1-\cos 2(4x)}{2} = \frac{1-\cos 8x}{2}$$

$$\int \sin^2 4x\, dx = \int \frac{1-\cos 8x}{2} = \frac{1}{2}\int 1-\cos 8x\, dx = \frac{1}{2}\left(x - \frac{1}{8}\sin 8x\right) + C = \boxed{\frac{1}{2}x - \frac{1}{16}\sin 8x + C}$$

12. Evaluate using $\sin x = \frac{2u}{1+u^2}$, $\cos x = \frac{1-u^2}{1+u^2}$ and $dx = \frac{2}{1+u^2}du$ where $u = \tan\frac{x}{2}$.

Evaluate $\int \frac{1}{\tan x + \sin x}dx$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{2u}{1+u^2}}{\frac{1-u^2}{1+u^2}} = \frac{2u}{1+u^2}\cdot\frac{1+u^2}{1-u^2} = \frac{2u}{1-u^2}$$

$$\tan x + \sin x = \frac{2u}{1-u^2} + \frac{2u}{1+u^2} = \frac{2u(1+u^2)+2u(1-u^2)}{(1-u^2)(1+u^2)} = \frac{2u+2u^3+2u-2u^3}{(1-u^2)(1+u^2)} = \frac{4u}{(1-u^2)(1+u^2)}$$

$$\int \frac{1}{\tan x + \sin x}dx = \int \frac{1}{\frac{4u}{(1-u^2)(1+u^2)}}\frac{2}{1+u^2}du = \int \frac{(1-u^2)(1+u^2)}{4u}\frac{2}{1+u^2}du = \int \frac{(1-u^2)}{2u}du$$

$$= \int \left[\frac{1}{2u} - \frac{u^2}{2u}\right]du = \int \left[\frac{1}{2u} - \frac{u}{2}\right]du = \tfrac{1}{2}\ln|u| - \tfrac{1}{4}u^2 + C = \boxed{\tfrac{1}{2}\ln\left|\tan\tfrac{x}{2}\right| - \tfrac{1}{4}\tan^2\tfrac{x}{4} + C}$$

13. Evaluate $\int \frac{1}{\sqrt[4]{x} + \sqrt[3]{x}}dx$

Let $u = x^{\frac{1}{12}}$, $u^{12} = x$, $12u^{11} = dx$

$$\int \frac{1}{\sqrt[4]{u^{12}} + \sqrt[3]{u^{12}}}\cdot 12u^{11}du = 12\int \frac{u^{11}}{u^3+u^4}\cdot du = 12\int \frac{u^8}{1+u}\cdot du =$$

Using Long Division…

$$\frac{u^8}{1+u} = u^7 - u^6 + u^5 - u^4 + u^3 - u^2 + u - \frac{1}{1+u}$$

$$12\int \frac{u^8}{1+u}\cdot du = 12\int \left[u^7 - u^6 + u^5 - u^4 + u^3 - u^2 + u - \frac{1}{1+u}\right]\cdot dx$$

$$= 12\left(\tfrac{1}{8}u^8 - \tfrac{1}{7}u^7 + \tfrac{1}{6}u^6 - \tfrac{1}{5}u^5 + \tfrac{1}{4}u^4 - \tfrac{1}{3}u^3 + \tfrac{1}{2}u^2 - \ln|1+u|\right) + C$$

$$\boxed{= \tfrac{4}{3}x^{\frac{3}{4}} - \tfrac{12}{7}x^{\frac{7}{12}} + 2x^{\frac{1}{2}} - \tfrac{12}{5}x^{\frac{5}{12}} + 3x^{\frac{1}{3}} - 4x^{\frac{1}{4}} + 6x^{\frac{1}{6}} - 12\ln\left|1+x^{\frac{1}{12}}\right| + C}$$