5. Evaluate $\int \frac{3x^2-6x+9}{(x^2+9)(x-3)}\,dx$

Partial Fractions

$$\frac{3x^2-6x+9}{(x^2+9)(x-3)} = \frac{Ax+B}{x^2+9} + \frac{C}{x-3} \quad \Rightarrow \quad 3x^2-6x+9 = (Ax+B)(x-3) + C(x^2+9)$$

$$3x^2-6x+9 = Ax^2 - 3Ax + Bx - 3B + Cx^2 + 9C = (A+C)x^2 + (B-3A)x + (-3B+C)$$

$$A + C = 3$$
$$-3A + B = -6$$
$$-3B + 9C = 9$$

Notice If $x = 3$, then $3(3)^2 - 6(3) + 9 = (A(3)+B)(3-3) + C(3^2+9)$

$$27 - 18 + 9 = C(9+9)$$
$$18 = 18C \quad \Rightarrow \quad \mathbf{C = 1}$$

If $C = 1$, then $A + 1 = 3 \quad \Rightarrow \quad \mathbf{A = 2}$

If $A = 2$, then $-3(2) + B = -6 \quad \Rightarrow \quad \mathbf{B = 0}$

$$\int \frac{3x^2-6x+9}{(x^2+9)(x-3)}\,dx = \int \frac{2x}{x^2+9} + \frac{1}{x-3}\,dx = \boxed{\ln\left|x^2+9\right| + \ln\left|x-3\right| + C}$$

6. Evaluate $\int \frac{1}{x^2-2x+2}\,dx$

Complete Square

$$\int \frac{1}{x^2-2x+2}\,dx = \int \frac{1}{x^2-2x+1-1+2}\,dx = \int \frac{1}{(x-1)^2+1}\,dx$$

Let $u = x - 1$

$du = dx$

$$\int \frac{1}{u^2+1}\,dx = \frac{1}{1}\tan^{-1}\frac{u}{1} + C = \boxed{\tan^{-1}(x-1) + C}$$

Find the limit if it exists and state the form of indeterminate for each part.

7. $$\lim_{x\to\infty}\frac{x^2-1}{e^x+1}=\lim_{x\to\infty}\frac{2x}{e^x}=\lim_{x\to\infty}\frac{2}{e^x}=\boxed{\infty}$$

Indeterminate form $\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

Use L'Hôpital's rule twice

8. $$\lim_{x\to 1}\frac{\ln x}{x-1}=\lim_{x\to 1}\frac{\frac{1}{x}}{1}=\lim_{x\to 1}\frac{1}{x}=\frac{1}{1}=\boxed{1}$$

Indeterminate form $\frac{0}{0}$

Use L'Hôpital's rule once

9. $$\lim_{x\to 0^+} x^x$$

Use this method

$y=x^x$

$\ln y=\ln x^x$

$\ln y=x\ln x$

$$\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}x\ln x=\lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x}}=\lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}}=\lim_{x\to 0^+}\frac{-x^2}{x}=\lim_{x\to 0^+}-x=-0=0$$

Indeterminate form $0\cdot\infty$, but now $\frac{\infty}{\infty}$. Use L'Hôpital's rule once

$$\lim_{x\to 0^+}\ln y=\ln\lim_{x\to 0^+}x^x=0\Rightarrow\lim_{x\to 0^+}x^x=e^0=\boxed{1}$$