

1. Evaluate $\int \sin^3 x \, dx$

Let $u = \cos x$
 $du = -\sin x \, dx$

$$\int \sin^3 x \, dx = \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \sin x \, dx = -\int (1 - u^2) \, du$$

$$\int (-1 + u^2) \, du = -u + \frac{1}{3}u^3 + C = \boxed{-\cos x + \frac{1}{3}\cos^3 x + C}$$

2. Evaluate $\int_0^{\frac{\pi}{4}} \tan^3 x \, dx$

$$\int \tan^3 x \, dx = \int \tan^2 x \cdot \tan x \, dx = \int (\sec^2 x - 1) \tan x \, dx = \int (\sec^2 x \cdot \tan x - \tan x) \, dx$$

$$= \int (\sec^2 x \cdot \tan x) \, dx - \int \tan x \, dx$$

First Integral

$$\int \sec^2 x \cdot \tan x \, dx = \int u \cdot du = \frac{1}{2}u^2 + C_1 = \frac{1}{2}\tan^2 x + C_1$$

Let $u = \tan x$
 $du = \sec^2 x \, dx$

Second Integral

$$\int \tan x \, dx = -\ln|\cos x| + C_2$$

Put it all together

$$\int_0^{\frac{\pi}{4}} \sec^2 x \tan x \, dx - \int_0^{\frac{\pi}{4}} \tan x \, dx = \left. \frac{1}{2}\tan^2 x - (-\ln|\cos x|) \right|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\tan^2 x + \ln|\cos x| \Big|_0^{\frac{\pi}{4}} = \frac{1}{2}\tan^2 \frac{\pi}{4} + \ln\left|\cos \frac{\pi}{4}\right| - \left(\frac{1}{2}\tan^2 0 + \ln|\cos 0|\right)$$

$$= \frac{1}{2}(1) + \ln \frac{\sqrt{2}}{2} - 0 - 0 = \boxed{\frac{1}{2} + \ln \frac{\sqrt{2}}{2}}$$

$$\frac{1}{2} + \ln\left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{1}{2} + \frac{1}{2}\ln\left(\frac{1}{2}\right) = \frac{1 + \ln \frac{1}{2}}{2}$$

Ti-89 Calculator Answer

3. Evaluate $\int \frac{\sin^2 x}{\cos x} dx$

$$\int \frac{1 - \cos^2 x}{\cos x} dx = \int \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} dx = \int \sec x - \cos x dx$$

$$= \ln|\sec x + \tan x| - \cos x + C$$

4. Evaluate $\int \frac{-x+1}{2x^2+x} dx$

Partial Fractions

$$\frac{-x+1}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1} \Rightarrow -x+1 = A(2x+1) + Bx$$

If $x = 0$

$$-0+1 = A(2 \cdot 0 + 1) + B(0)$$

$$1 = A$$

If $x = -\frac{1}{2}$

$$-(-\frac{1}{2}) + 1 = A(2(-\frac{1}{2}) + 1) + B(-\frac{1}{2})$$

$$\frac{3}{2} = -\frac{1}{2}B \Rightarrow B = -3$$

$$\int \frac{-x+1}{2x^2+x} dx = \int \frac{1}{x} - \frac{3}{2x+1} dx = \ln|x| - 3 \cdot \frac{1}{2} \ln|2x+1| + C = \ln|x| - \frac{3}{2} \ln|2x+1| + C$$

REMARK: $\int \frac{3}{2x+1} dx$ Let $u = 2x+1$
 $du = 2dx$
 $\frac{1}{2} du = dx$

$$\int \frac{3}{2x+1} dx = 3 \cdot \frac{1}{2} \int \frac{1}{u} du = \frac{3}{2} \ln|u| + C = \frac{3}{2} \ln|2x+1| + C$$