Math 182 Test 3 Name ______KEY__________

McDonald Date 4/19/2004

7 Points each

1. Evaluate $\int \sin^3 x \; dx$

Let $\begin{aligned} u &= \cos x \\ du &= -\sin x \, dx \end{aligned}$

$$\int \sin^3 x \; dx = \int \sin^2 x \cdot \sin x \; dx = \int (1-\cos^2 x)\sin x \; dx = -\int (1-u^2)\, du$$

$$\int (-1+u^2)\, du = -u + \tfrac{1}{3}u^3 + C = \boxed{-\cos x + \tfrac{1}{3}\cos^3 x + C}$$

2. Evaluate $\int_0^{\frac{\pi}{4}} \tan^3 x \; dx$

$$\int \tan^3 x \; dx = \int \tan^2 x \cdot \tan x \; dx = \int (\sec^2 x - 1)\tan x \; dx = \int (\sec^2 x \cdot \tan x - \tan x) \; dx$$

$$= \int (\sec^2 x \cdot \tan x) \; dx - \int \tan x \; dx$$

First Integral

$$\int \sec^2 x \cdot \tan x \; dx = \int u \cdot du = \tfrac{1}{2}u^2 + C_1 = \tfrac{1}{2}\tan^2 x + C_1$$

Let $\begin{aligned} u &= \tan x \\ du &= \sec^2 x \, dx \end{aligned}$

Second Integral

$$\int \tan x \; dx = -\ln|\cos x| + C_2$$

Put it all together

$$\int_0^{\frac{\pi}{4}} \sec^2 x \tan x dx - \int_0^{\frac{\pi}{4}} \tan x dx = \tfrac{1}{2}\tan^2 x - \left(-\ln|\cos x|\right)\Big]_0^{\frac{\pi}{4}}$$

$$= \tfrac{1}{2}\tan^2 x + \ln|\cos x|\Big]_0^{\frac{\pi}{4}} = \tfrac{1}{2}\tan^2 \tfrac{\pi}{4} + \ln\left|\cos \tfrac{\pi}{4}\right| - \left(\tfrac{1}{2}\tan^2 0 + \ln|\cos 0|\right)$$

$$= \tfrac{1}{2}(1) + \ln\tfrac{\sqrt{2}}{2} - 0 - 0 = \boxed{\tfrac{1}{2} + \ln\tfrac{\sqrt{2}}{2}}$$

$$\frac{1}{2} + \ln\left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{1}{2} + \frac{1}{2}\ln\left(\frac{1}{2}\right) = \frac{1 + \ln\frac{1}{2}}{2}$$

Ti-89 Calculator Answer

3. Evaluate $\int \frac{\sin^2 x}{\cos x}\, dx$

$$\int \frac{1-\cos^2 x}{\cos x}\, dx = \int \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}\, dx = \int \sec x - \cos x \; dx$$

$$\boxed{= \ln\left|\sec x + \tan x\right| - \cos x + C}$$

4. Evaluate $\int \frac{-x+1}{2x^2+x}\, dx$

Partial Fractions

$$\frac{-x+1}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1} \Rightarrow -x+1 = A(2x+1) + Bx$$

If $x = 0$

$$-0+1 = A(2\cdot 0+1) + B(0)$$
$$1 = A$$

If $x = -\frac{1}{2}$

$$-(-\tfrac{1}{2}) + 1 = A\left(2(-\tfrac{1}{2})+1\right) + B(-\tfrac{1}{2})$$
$$\tfrac{3}{2} = -\tfrac{1}{2}B \Rightarrow B = -3$$

$$\int \frac{-x+1}{2x^2+x}\, dx = \int \frac{1}{x} - \frac{3}{2x+1}\, dx = \ln|x| - 3\cdot\tfrac{1}{2}\ln|2x+1| + C = \boxed{\ln|x| - \tfrac{3}{2}\ln|2x+1| + C}$$

REMARK: $\int \frac{3}{2x+1}dx$ Let

$$u = 2x+1$$
$$du = 2dx$$
$$\tfrac{1}{2}du = dx$$

$$\int \frac{3}{2x+1}dx = 3\cdot\tfrac{1}{2}\int \frac{1}{u}du = \tfrac{3}{2}\ln|u| + C = \tfrac{3}{2}\ln|2x+1| + C$$