Math 182 Test 2 Name ____________________________
McDonald Date ________________
8 *Points each*

1. Find the derivative of $f(x) = 9^{x^2+2x}$

Note: use Chain rule.

$$f'(x) = (2x+2)\cdot 9^{x^2+2x}\ln x$$

2. Find the derivative of $y = e^{-\frac{3}{x^2}}$

$$y' = e^{-\frac{3}{x^2}}\left(\frac{6}{x^3}\right) = \frac{6e^{-\frac{3}{x^2}}}{x^3}$$

$$D_x\left[-\frac{3}{x^2}\right] = D_x\left[-3x^{-2}\right] = 6x^{-3} = \frac{6}{x^3}$$

Note: use Chain rule.

3. Use implicit differentiation to find y'

$xe^y + 2x - \ln(y+1) = 3$

$$(1)e^y + xe^y y' + 2 - \frac{y'}{y+1} = 0$$

$$xe^y y' - \frac{y'}{y+1} = -e^y - 2$$

$$\left(xe^y - \frac{1}{y+1}\right)y' = -e^y - 2$$

$$y' = \frac{-e^y - 2}{xe^y - \frac{1}{y+1}}$$

$$y' = \frac{\left(-e^y - 2\right)(y+1)}{\left(xe^y - \frac{1}{y+1}\right)(y+1)}$$

$$y' = -\frac{\left(e^y + 2\right)(y+1)}{xe^y(y+1) - 1}$$

4. Evaluate $\int \csc 2x dx$

let $u = 2x$

$du = 2\ dx$

$\frac{1}{2} du = dx$

$$\frac{1}{2}\int \csc u\ du = \frac{1}{2}\ln|\csc u - \cot u| + c$$

$$\boxed{\int \csc 2x dx = \frac{1}{2}\ln|\csc 2x - \cot 2x| + c}$$

5. Evaluate $\int_1^2 \frac{3x}{x^2+1} dx$

If $x = 1$, then $u = 1^2 + 1 = 2$
If $x = 2$, then $u = 2^2 + 1 = 5$

Let $u = x^2 + 1$
$du = 2x\ dx$

$$\frac{1}{2}\int \frac{3}{u} du = \frac{3}{2}\ln|u| + c$$

$$\int_1^2 \frac{3x}{x^2+1} dx = \frac{3}{2}\ln\left(x^2+1\right)\Big|_1^2 = \frac{3}{2}\left[\ln\left(2^2+1\right) - \ln\left(1^2+1\right)\right]$$

$$= \frac{3}{2}\left[\ln(5) - \ln(2)\right] = \boxed{\frac{3}{2}\left[\ln\frac{5}{2}\right]}$$

6. Evaluate $\int \frac{5^x}{5^x+1} dx$

Let $u = 5^x + 1$

$du = 5^x \ln 5\ dx$

$$\int \frac{5^x}{5^x+1} dx = \frac{1}{\ln 5}\int \frac{1}{u} du = \frac{1}{\ln 5}\cdot \ln|u| + c = \boxed{\frac{\ln\left(5^x+1\right)}{\ln 5} + c}$$

7. Find $f'(x)$ if $f(x) = \sin^{-1}(3x+5)$

$$f'(x) = \frac{3}{\sqrt{1-(3x+5)^2}}$$

8. Evaluate $\int \frac{x}{\sqrt{16-x^2}}dx$

Let $u = 16 - x^2$
$du = -2x\, dx$

$$-\frac{1}{2}\int \frac{1}{\sqrt{u}}\ dx = -\frac{1}{2}\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\ dx = -\frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = -\sqrt{16-x^2}+c$$

9. Evaluate $\int \frac{7}{x\sqrt{x^2-9}}dx$

$$\int \frac{7}{x\sqrt{x^2-9}}dx = \tfrac{7}{3}\sec^{-1}\tfrac{x}{3}+c$$

10. Evaluate $\int x^2 e^{-2x} dx$

Sign	u	Dv
+	x^2	e^{-2x}
−	$2x$	$-\frac{1}{2}e^{-2x}$
+	2	$\frac{1}{4}e^{-2x}$
		$-\frac{1}{8}e^{-2x}$

$$-\tfrac{1}{2}x^2e^{-2x}-\tfrac{2}{4}xe^{-2x}-\tfrac{2}{8}xe^{-2x}=\boxed{-\tfrac{1}{2}x^2e^{-2x}-\tfrac{1}{2}xe^{-2x}-\tfrac{1}{4}xe^{-2x}}$$

or $-\frac{1}{4}e^{-2x}(2x^2+2x-1)$

11. Evaluate (Must show) $\int \ln x \, dx$

Let $u=\ln x$ $\qquad dv = dx$

$du=\dfrac{1}{x}dx$ $\qquad v = x$

$$x\ln x-\int x\left(\frac{1}{x}\right)dx = x\ln x-\int 1\,dx=\boxed{x\ln x - x + c}$$

(Not Counted)

12. Evaluate $\int \sin^2 x dx$

$$\int \sin^2 x dx = \int \frac{1-\cos 2x}{2} dx = \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx = \boxed{\frac{1}{2}x - \frac{1}{4}\sin 2x + c}$$

13. Evaluate $\int_0^\pi \sin^3 x \cos^2 x\, dx$

$$\int_0^\pi \sin^3 x \cos^2 x\, dx = \int_0^\pi \sin^2 x \sin\ x \cos^2 x\, dx = \int_0^\pi \left(1-\cos^2 x\right)\sin\ x \cos^2 x\, dx$$

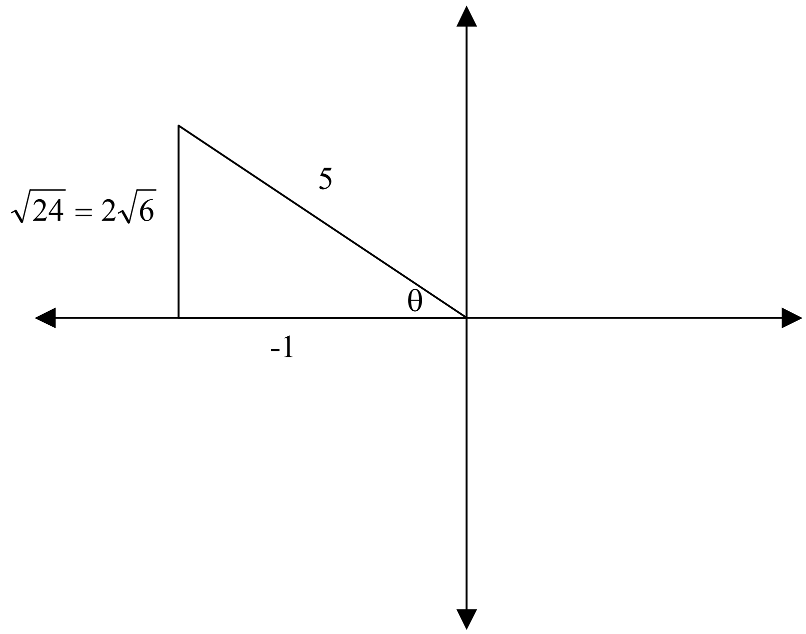
$$\text{Let}\quad \begin{aligned} u &= \cos x \\ du &= -\sin x\, dx \\ -du &= \sin x dx \end{aligned}$$

$$-\int \left(1-u^2\right)u^2\ dx = -\int \left(u^2 - u^4\right) du = -\frac{1}{3}u^3 + \frac{1}{5}u^5 = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x$$

$$-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x\Big|_0^\pi = -\frac{1}{3}(-1)^3 + \frac{1}{5}(-1)^5 - \left(-\frac{1}{3}(1)^3 + \frac{1}{5}(1)^5\right) = \frac{1}{3} - \frac{1}{5} + \frac{1}{3} - \frac{1}{5} = \boxed{\frac{4}{15}}$$

14. Find $\csc\left[\cos^{-1}\left(-\frac{1}{5}\right)\right]$ exactly.

$$\theta = \cos^{-1}\left(\frac{-1}{5}\right) \Rightarrow \cos\theta = \frac{adj}{hyp} = -\frac{1}{5}$$



$$\csc\theta = \frac{hyp}{opp} = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \boxed{\frac{5\sqrt{6}}{12}}$$

Extra Credit: No Calculators Either!!!

Find $\int \frac{1}{x\sqrt{x-1}}\, dx$

Let
$$u = \sqrt{x} \Rightarrow u^2 = x$$
$$du = \frac{1}{2\sqrt{x}}\, dx$$
$$2du = \frac{1}{\sqrt{x}}\, dx$$

$$\int \frac{1}{x\sqrt{x-1}}\, dx = \int \frac{1}{\sqrt{x}\cdot\sqrt{x}\sqrt{x-1}}\, dx = 2\int \frac{1}{u\cdot\sqrt{u^2-1}}\, du$$

$$\boxed{= 2\sec^{-1} u + c = 2\sec^{-1}\sqrt{x} + c =}$$